Who Is Shown to Whom? Sorting, Screening, and Exposure Design on a Two-Sided Matching Platform*

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27th November 2026

Abstract

I study how platforms should design recommendations when users also search independently. Building on Adachi (2003), I develop a frictional two-sided search model with two exposure routes, users' own search and platform recommendations. Users choose how intensively to search, and a match requires acceptance by both sides. I estimate the model using exposure and bilateral acceptance data from a Japanese platform matching doctors to short-term jobs. Using the lattice structure of the model's equilibria, I verify that the estimated market and the optimized designs each have a unique equilibrium. The counterfactuals favor recommendations that complement doctor search and prioritize long-run value. Retaining search makes welfare and the distribution of benefits across posts less sensitive to the recommendation objective, while long-run targeting improves both immediate surplus and long-run value relative to existing recommendations, albeit with uneven gains. The value of redesign also persists as recommendations expand: reallocating recommendations across posts yields a larger gain than doubling recommendation volume and equalizing totals across doctors combined.

JEL Classification Codes: D47; D83; C78; J64; C61; C63; L86.

Keywords: two-sided platforms; sequential search; sorting; screening; intermediation; market design; recommendation systems; doctor–job matching.

*I am grateful to Medical Principle Co. for providing the data and for their thoughtful support in helping me understand the institutional background. I thank Yu Awaya, Michihiro Kandori, Fuhito Kojima, Kyohei Okumura, and Kosuke Uetake for valuable comments. I also thank participants at the ERATO meeting, IIOC, and SSCW for helpful feedback. This work was supported by JSPS KAKENHI Grant Number JP26K16347. Earlier versions of this paper circulated under the title "Exposure Design for Two-Sided Platforms."

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1 Introduction

Two-sided matching platforms rarely choose matches directly. They decide who is shown to whom. A match begins with an *exposure*, an event in which a participant on one side is shown to a participant on the other and both then decide whether to proceed. Exposures arise through two routes. On the *sorting* route, participants search the listings themselves and choose whom to approach. On the *screening* route, the platform acts on behalf of one side, selects a candidate from the other side, and recommends the match to that candidate. A recommendation policy therefore operates alongside participants' own search rather than in isolation.

Designing recommendations is difficult because participants respond to them. After an exposure, either side can decline, and a participant with better prospects declines more often. Before an exposure, a participant with better prospects searches less. Because of these responses, creating matches immediately, creating value in the long run, and spreading matching opportunities across the market are different goals. A rule that maximizes one of them can do poorly on the others.

A platform that designs a recommendation algorithm therefore faces three questions. The first is whether recommendations should complement participants' own search or replace it. Replacing search gives the platform full control over exposures, but it also removes the margin along which participants adjust to the recommendations. The second is what the algorithm should prioritize: immediate expected surplus, long-run value, or broad access to matching opportunities. These goals conflict, so the answer depends on how much of each the platform gives up when it pursues another. The third is whether who receives recommendations matters as much as how many are made. A platform can also raise value by making more recommendations or by attracting more participants, and it needs to know how these gains compare with the gain from reallocating the recommendations it already makes. This paper answers these three questions for a platform on which participants respond by changing how intensively they search.

I answer these questions in two steps. First, I build a frictional two-sided search model in the tradition of Adachi (2003). In the model, exposures arrive through the sorting and screening routes, both sides decide whether to accept after each exposure, and doctors choose how intensively to search. I show that the equilibria of this model form a lattice, and I use this structure to verify that the equilibrium is unique in the market I study. Second, I estimate the model using data from a Japanese platform where doctors apply to short-term posts and platform agents recommend posts to doctors. The data identify the route that initiated each exposure and record each side's decision to accept or reject. The platform is considering an algorithmic recommendation system, so I use the estimated model to compare recommendation policies while doctor search adjusts.

The comparisons yield three findings. First, doctor search acts as a buffer. When recommendations change, doctors adjust how intensively they search, and this response offsets part of the change. With doctor search retained, short- and long-run welfare and the distribution of benefits across posts are therefore less sensitive to which objective the recommendations maximize. Second, with doctor search retained, the platform should prioritize long-run value, the total value that doctors and posts expect from the platform once search and acceptance have adjusted to the recommendations. The *long-run rule*, which allocates recommendations to maximize this value, raises both immediate expected surplus and long-run value relative to the platform's existing recommendations, although the gains are uneven across posts. Third, reallocating recommendations remains valuable when the platform can make more of them. Doubling recommendation volume and equalizing totals across doctors raise value, but the subsequent redesign across posts adds more value than those two changes combined. That final gain is comparable to adding about 2 percent more doctors or 6 percent more posts.

Related literature. The paper first contributes to the frictional matching literature on sorting and screening (Eeckhout and Kircher, 2010; Moen, 1997; Burdett and Coles, 1997; Shimer and Smith, 2000; Chade, Eeckhout and Smith, 2017) by making the contrast between doctor-initiated search and platform-initiated screening empirically operational. The same platform contains both routes, the data identify which route generated each exposure, and the model evaluates alternative channel architectures and allocation rules for a given number of platform-assigned exposures.

On the theoretical side, Section 4 extends the order structure of decentralized two-sided search established by Adachi (2003). The extension allows for two exposure routes, doctors who search for several jobs at once, exposure rates chosen by the participants, and exposure rates set by the platform. The section uses this structure to verify that the estimated equilibrium is unique, even though such markets can have several equilibria (Burdett and Wright, 1998).

Second, the paper relates to empirical work on online matching platforms (Hitsch, Hortaçsu and Ariely, 2010; Fradkin, 2017; Gong, 2016; Agarwal, 2015; Galichon, Kominers and Weber, 2019). Its distinctive feature is identifying the initiating route and observing doctor- and post-side acceptance or rejection decisions, rather than only realized matches.

Third, the paper connects to work on recommendations and steering in labor and platform markets (Horton, 2017; Belot, Kircher and Muller, 2019; Le Barbanchon, Hensvik and Rathelot, 2023; Chen and Tsai, 2024), which estimates the effects of providing or changing recommendations. Here recommendations are one route inside a structural two-route system, so the counterfactuals compare alternative channel architectures and allocation rules on a common footing.

Finally, the paper contributes to algorithmic and operations work on search design, mul-

tichannel traffic, fairness, and two-sided recommendation (Immorlica et al., 2023; Manshadi et al., 2025; Shi, 2023, 2025; Chen, Hsieh and Lin, 2023; Sürer, Burke and Malthouse, 2018; Biega, Gummadi and Weikum, 2018): the recommendation rule is evaluated through equilibrium continuation values under finite doctor capacity rather than through immediate conversion or ranking metrics alone.

2 Setting and Data

2.1 Platform and Exposure Channels

The empirical setting is a Japanese two-sided labor platform operated by Medical Principle Co., Ltd. The platform matches licensed doctors with short-term "spot" positions at hospitals and clinics. Participating doctors typically hold regular positions elsewhere. Hospitals and clinics post temporary openings, primarily for health checkups, ward coverage, and overnight on-call shifts. The platform records the applications and recommendations associated with each opening, as well as the subsequent decisions by both sides that lead to a match.

A doctor and a post can be brought together through either of two routes. A doctor may search the platform and apply to a post; I refer to this route as *doctor-initiated search*, or the *sorting route*. Alternatively, a platform agent acting on behalf of a post may select a doctor and recommend the post to that doctor; I refer to this route as *agent-initiated screening*, or the *screening route*. Medical institutions do not directly search the platform's doctor pool. Under the platform's current operation, doctor-initiated search therefore reflects information and initiative on the doctor side, whereas agent-initiated screening is the route that the platform can directly redesign.[1]

I use *exposure* to refer to the recorded event on either route: an application on the sorting route or a recommendation on the screening route. An exposure is distinct from a listing view: it brings a particular doctor–post pair together and initiates an exchange of detailed information through the intermediary, often in the form of an interview. Both sides then decide whether to proceed, and a match forms only if both sides accept. A doctor who applied may still decline after learning the details, and either side may reject a recommendation initiated by an agent.

2.2 Market and Measurement

The online job-matching platform operates continuously, so the empirical analysis must isolate a specific market from the ongoing flow of doctors and posts. This subsection first explains how I define the estimation cohort. It then describes how I construct bilateral acceptance decisions

[1]The platform's user interface is itself a potential object of design. For example, the platform could remove the search function available to doctors and instead generate all exposures through algorithmic recommendations. Section 6 considers this more substantial change to platform functionality as a counterfactual benchmark.

and predetermined measures of prior platform activity within that cohort, before presenting its main descriptive patterns.

Estimation cohort. The estimation cohort consists of the posts in the Kanto region whose scheduled work date falls in December 2024, together with every doctor who has at least one exposure to one of these posts. I follow each post from the day the platform receives its order until the day its contract is concluded or, if the post is not filled, until its scheduled work date. Every exposure to the post during this period enters the analysis, including exposures that occur before December. The final cohort contains 1,132 doctors and 2,412 posts. All other doctors on the platform, who neither apply to one of these posts nor receive a recommendation for one, are outside the cohort. I regard them as *inactive* in the December 2024 spot-job market, because they generate no recorded indication that they are considering one of its posts.

The data also contain basic characteristics of both sides. Doctor characteristics include age, experience, specialty, location, and stated preferences. Post characteristics include location, task content, desired specialty, schedule, working hours, and compensation. From these characteristics, I also construct three indicators that are specific to a doctor–post pair. The preferred-location indicator records whether the post lies in the doctor's preferred prefecture or region. The work-schedule indicator, labeled "Day match" in the tables, records whether the post matches a preferred combination of weekday and daytime or overnight work.[2] The specialty indicator records whether the doctor has at least one specialty in the post's specialty list.

Route and decision construction. The data record the initiating route and subsequent status of each exposure, from which I construct separate doctor- and post-side acceptance indicators. For each exposed doctor–post pair, the platform records one of five statuses: *Contract*, *Cancelled After Contract*, *Not Hired*, *Approach*, or *Inquiry Handled*. The mapping from these statuses to binary acceptance indicators, constructed in consultation with the platform's staff, is reported in Table 1.

Past platform activity. I construct five predetermined variables separately for each doctor–facility pair, using only activity recorded on the platform strictly before the order date of the current post: the number of exposures between the doctor and the facility during the preceding 365 days; an indicator for any prior contract between the doctor and the facility; the number of days since the pair's most recent exposure; the share of the pair's exposures during the preceding 365 days that were initiated by an agent; and an indicator that no prior relationship between

[2]The post's shift category is constructed from job duration: posts longer than ten hours are classified as overnight and the remaining posts as daytime.

Table 1. Mapping from Recorded Status to Acceptance Indicators

Recorded status	Doctor accepts	Post accepts
Contract	1	1
Cancelled After Contract	1	1
Not Hired	1	0
Approach	0	1
Inquiry Handled	0	–

Notes: 1 denotes acceptance, 0 denotes rejection, and – denotes an undefined decision. An undefined decision is treated as missing rather than as a rejection.

the doctor and the facility is observed. These variables capture persistent familiarity in the four empirical indices.

The model also allows doctors to differ in how many posts they can search simultaneously. I interpret this latent quantity as an activity or throughput measure, motivated by the substantial differences in recorded platform activity among otherwise observably similar licensed doctors. Because this quantity is not directly observed, I proxy it with the maximum, over September–November 2024, of a doctor's monthly number of recorded exposures across both routes, excluding exposures to posts in the December cohort and imposing a floor of one slot.

Descriptive patterns. A notable feature of the data is the sharp asymmetry according to who initiates an exposure. Among observed decisions following doctor-initiated search, doctors accept 98.0 percent and posts accept 41.0 percent. Following agent-initiated screening, the corresponding acceptance rates are 28.3 percent for doctors and 98.4 percent for posts. Thus, doctor-initiated search reflects substantial doctor-side preselection, whereas agent-initiated screening reflects substantial post-side preselection.

Controlling for past doctor–facility activity is particularly important when estimating these acceptance patterns. Only 121,065 of the 2,730,384 possible doctor–post pairs, or 4.43 percent, have a nonzero prior doctor–facility history. By contrast, among the 3,281 doctor–post pairs whose recorded decision can be attributed to a single route, and which therefore enter the acceptance likelihood, 1,685 pairs, or 51.36 percent, have such history. Pairs with an observed acceptance decision are therefore much more likely than an average possible doctor–post pair to involve a doctor and facility with a pre-existing relationship. Without these recorded-history variables, differences associated with prior familiarity could be attributed to the initiating route or to contemporaneous pair characteristics. Appendix B.2 presents separate summary tables for past activity and other analysis variables.

3 Model

I develop a continuous-time variant of the two-sided search model of Adachi (2003). The two sides of the market are doctors and posts. On the doctor side, I call the unit that can form a match a *doctor slot*. A doctor may operate several interchangeable slots, which allows the model to represent differences in how actively doctors search and, in particular, how many spot jobs they consider concurrently. Observed characteristics and preferences are shared across slots belonging to the same doctor, but each slot is treated as an agent in the model. No analogous construction is needed on the post side: because an open post leaves the market once it matches, each post is itself one model agent.

The sequence of events in continuous time is as follows. At any instant, each unmatched doctor slot and each open post is in its own waiting state and has a continuation value that summarizes its future matching opportunities. For each doctor–post pair, the doctor chooses the rate of doctor-initiated search, while a platform agent acting on behalf of the post chooses the rate of agent-initiated screening. These pair- and route-specific rates govern independent Poisson clocks. When one of these clocks rings, an exposure occurs. The doctor and post then observe the information revealed by that exposure and independently decide whether to proceed. If both accept, a match forms and the matched doctor slot and post exit. If either rejects, the doctor slot remains unmatched and the post remains open, so their exposure processes continue.

The remainder of the section follows this order: I first describe the choice of exposure rates and the arrival of exposures, then acceptance after an exposure, then continuation values and stationary renewal.

3.1 Agents, Timing, and Exposure Decisions

Agents and exposure routes. Time is continuous. Let I be the set of doctors, J the set of posts, and $\mathcal{C} = \{\text{self}, \text{agency}\}$ the two exposure routes. The route *self* is the sorting route of Section 2: a doctor-initiated exposure generated by the doctor's search over listings. The route *agency* is the screening route: an agent-initiated recommendation generated by screening on behalf of the post. Doctor i has $K_i \geq 1$ interchangeable doctor slots. Each post j is one open position and therefore one model agent. Each doctor slot can receive exposures through both routes.

Choosing exposure rates. For each doctor–post pair, the initiator of each route chooses the instantaneous rate at which that route generates an exposure. On the self route, a higher rate represents more search attention by the doctor toward the post and therefore a higher arrival hazard of an application. On the agency route, it represents more screening attention directed toward the pair and therefore a higher arrival hazard of a recommendation. I represent

this decision as a pair- and route-specific choice of a Poisson arrival rate. The rate determines both the expected number of exposures per unit of time and the instantaneous hazard that an exposure occurs. The initiator chooses the rate by comparing the gross value of an exposure with the value of waiting.

Let s_{ij}^c denote the route-specific gross value that the initiator assigns to generating one additional exposure between doctor i and post j. This is a pre-exposure value index. It is distinct from the match payoff that the two sides observe after an exposure occurs and from the total-user-value objective used for policy evaluation. The value of waiting is the continuation value of the initiating side:

$$\omega_{ij}^c(\alpha) = \begin{cases} \alpha_i^D, & c = \text{self}, \\ \alpha_j^P, & c = \text{agency}, \end{cases}$$

where α_i^D is the value of keeping one slot of doctor i available for future matching opportunities, and α_j^P is the value of keeping post j open. The stationary equilibrium defined below requires them to coincide with the values generated by the resulting exposure-arrival and acceptance processes.

Moving the exposure rate away from a route-specific reference rate incurs an adjustment cost. For a Poisson rate $\lambda > 0$ and a route-specific reference rate $\bar{\lambda}^c > 0$, define this cost as

$$D_{\mathrm{KL}}(\lambda \| \bar{\lambda}^c) = \lambda \log\left(\frac{\lambda}{\bar{\lambda}^c}\right) - \lambda + \bar{\lambda}^c.$$

This expression is the relative-entropy rate of the Poisson process with rate λ relative to the reference process with rate $\bar{\lambda}^c$. It is nonnegative, equals zero at the reference rate, and is strictly convex in λ. I use it as an auxiliary convex penalty for moving the search or screening process away from its baseline.[3]

The initiator's auxiliary choice problem in the current market is

$$\max_{\lambda > 0} \quad \lambda\big(s_{ij}^c - \omega_{ij}^c(\alpha)\big) - D_{\mathrm{KL}}(\lambda \| \bar{\lambda}^c). \tag{1}$$

The first term is the flow value of generating exposures, net of the initiator's value of waiting. The second term is the cost of moving the exposure rate away from its reference level. Its marginal cost is $\log(\lambda/\bar{\lambda}^c)$, so the objective is strictly concave and the unique conditional choice satisfies

$$\log \lambda_{ij}^c(\alpha) = \log \bar{\lambda}^c + s_{ij}^c - \omega_{ij}^c(\alpha). \tag{2}$$

A positive net value raises the rate above that baseline, while a negative net value lowers it. On the self route, the doctor makes this choice. On the agency route, I use the same rule for the platform agent screening on behalf of the post.

[3]Relative-entropy penalties for moving a controlled stochastic law away from a passive reference law are used in KL-control formulations; see Todorov (2006). Wang et al. (2017) applies this idea directly to the control of point-process intensities. Here the device only rationalizes the functional form of the exposure-rate equation: the penalty itself does not enter continuation values or the policy objective.

Given the chosen rates, exposures between each slot of doctor i and post j arrive through two independent Poisson processes, with rates $\lambda_{ij}^{\text{self}}$ and $\lambda_{ij}^{\text{agency}}$. The two clocks almost surely do not ring simultaneously, so their rates add without an overlap correction. Across doctor i's K_i slots, the aggregate route-c exposure rate to post j is $K_i\lambda_{ij}^c$.

Acceptance after an exposure. An exposure is not a match. It is an event at which the two sides observe their match-specific information and then make bilateral acceptance decisions. Conditional on route c, their one-time payoffs are

$$U_{ij}^c = u_{ijc}^D + \varepsilon_{ijc}^D, \tag{3}$$

$$V_{ji}^c = u_{jic}^P + \varepsilon_{jic}^P. \tag{4}$$

The shocks are independent logistic random variables with scales $\zeta^D, \zeta^P > 0$, drawn independently across sides and exposures. Utility is nontransferable, and the pair matches if and only if both sides accept. The deterministic terms u_{ijc}^D and u_{jic}^P may depend on the route.

The acceptance rules are

$$a_{ijc}^D = \mathbf{1}\{U_{ij}^c \geq \alpha_i^D\}, \qquad a_{jic}^P = \mathbf{1}\{V_{ji}^c \geq \alpha_j^P\}.$$

Let $a = (a_{ijc}^D, a_{jic}^P)_{i\in I,\, j\in J,\, c\in\mathcal{C}}$ collect these rules. Write the resulting route-specific acceptance probabilities as

$$P_{ijc}^D(\alpha) = \frac{1}{1+\exp\{(\alpha_i^D - u_{ijc}^D)/\zeta^D\}}, \qquad P_{jic}^P(\alpha) = \frac{1}{1+\exp\{(\alpha_j^P - u_{jic}^P)/\zeta^P\}}.$$

Hence, bilateral acceptance on route c occurs with probability $p_{ijc}(\alpha) = P_{ijc}^D(\alpha)P_{jic}^P(\alpha)$.

3.2 Stationary Renewal and Continuation Values

This subsection determines the continuation values α_i^D and α_j^P that the previous subsection took as given. I first state the assumption under which these values are constant over time and then derive the equations that they satisfy.

Assumption 1 (Stationary renewal)**.**

(i) (Time-invariant primitives) *Gross exposure values, baseline rates, payoff distributions, stocks, exit hazards, and discounting do not vary over time.*

(ii) (Renewal) *When a doctor slot exits through a match or abandonment, or a post exits through a match or withdrawal, it is immediately renewed by a unit with the same modelled type. Hence the type composition and exposure environment remain fixed.*

(iii) (Absorbing payoffs) *A matched unit receives its one-time payoff and exits; exogenous exit yields zero. The renewed unit starts a new search problem with independent future shocks.*

Let $\delta_i^a \geq 0$ be doctor i's exogenous abandonment hazard, $w_j \geq 0$ post j's withdrawal hazard, and $r > 0$ the discount rate. I model the market as stationary. In addition to leaving after a match, a doctor slot may become inactive and an open post may be withdrawn before matching. Either event removes that unit from the active market for reasons outside the model. Under stationary renewal, every exiting unit is immediately replaced by another unit of the same modeled type. Market stocks and the marginal type distributions on both sides therefore remain fixed. This is an analytical approximation, not a claim that the platform literally replaces each departing unit with an identical clone.[4] Because the empirical market is calibrated to a single monthly cohort, I hold the marginal type distributions fixed when comparing stationary policies.

For route c, define the expected own-side gain above continued waiting,

$$q_{ijc}^D(\alpha) = \mathbb{E}[(U_{ij}^c - \alpha_i^D)_+], \qquad q_{ijc}^P(\alpha) = \mathbb{E}[(V_{ji}^c - \alpha_j^P)_+],$$

where $x_+ = \max\{x, 0\}$. Because the two payoff shocks are independent, the expected gains from an exposure, taking the other side's acceptance into account, are

$$\psi_{ijc}^D(\alpha) = P_{jic}^P(\alpha) q_{ijc}^D(\alpha), \qquad \psi_{ijc}^P(\alpha) = P_{ijc}^D(\alpha) q_{ijc}^P(\alpha).$$

The route matters both through its exposure rate and through the deterministic post-exposure payoffs inside P, q, and ψ.

Balancing the exposure gains against discounting and exogenous exit gives the route-aware continuation-value system

$$\begin{aligned} \alpha_i^D = \Phi_{i,K}^D(\alpha; \lambda) &:= \frac{\sum_{j \in J} \sum_{c \in \mathcal{C}} \lambda_{ij}^c \psi_{ijc}^D(\alpha)}{r + \delta_i^a}, \\ \alpha_j^P = \Phi_{j,K}^P(\alpha; \lambda) &:= \frac{\sum_{i \in I} \sum_{c \in \mathcal{C}} K_i \lambda_{ij}^c \psi_{ijc}^P(\alpha)}{r + w_j}. \end{aligned} \tag{5}$$

The difference between the two equations reflects units: λ_{ij}^c is the exposure rate for one doctor slot, whereas $K_i \lambda_{ij}^c$ is the aggregate flow from doctor i to post j.

Let $\Phi_K(\alpha; \lambda)$ stack the right-hand sides of (5), and define

$$\mathcal{Z}(\lambda^{\text{self}}, \lambda^{\text{agency}}, K) = \{\alpha \in \mathbb{R}_{\geq 0}^{|I|+|J|} : \alpha = \Phi_K(\alpha; \lambda)\}.$$

The system (5) takes the exposure rates λ as given. In the current market, however, these rates are themselves chosen and therefore depend on the continuation values. Let $\Gamma(\alpha; s)$ stack the behavioral rate rules in equation (2). I define an equilibrium for each of the two cases.

Definition 1 (Stationary-renewal equilibrium)**.**

[4] Richer steady-state search models instead allow entry and matching jointly to determine the composition of the unmatched stocks; see, for example, Lauermann and Nöldeke (2025). Allowing the type distributions to evolve as units enter and leave the market would be substantially less tractable at the scale of this application.

(i) (Fixed rates) *Fix* $(\lambda^{\text{self}}, \lambda^{\text{agency}}, K)$. *A fixed-rate stationary-renewal equilibrium is a pair* (a, α) *such that each side uses its route-specific threshold rule after every exposure and* α *equals the expected discounted value of waiting generated by those rules under Assumption 1.*

(ii) (Behavioral rates) *A behavioral stationary-renewal equilibrium is a triple* (λ, a, α) *such that* $\lambda = \Gamma(\alpha; s)$ *and* (a, α) *is a fixed-rate stationary-renewal equilibrium at the rates* λ.

Proposition 1 (Characterisation and existence)**.** *Let* I *and* J *be finite.*

(i) For nonnegative finite route rates, the fixed-rate stationary-renewal equilibria are exactly the threshold rules generated by $\alpha \in \mathcal{Z}(\lambda^{\text{self}}, \lambda^{\text{agency}}, K)$*, and this set is nonempty.*

(ii) The behavioral stationary-renewal equilibria are exactly the triples (λ, a, α) *in which* $\alpha \geq 0$*, the pair* (λ, α) *solves*

$$\lambda = \Gamma(\alpha; s), \qquad \alpha = \Phi_K(\alpha; \lambda), \tag{6}$$

and a *consists of the threshold rules generated by* α*. At least one such triple exists.*

Proof. See Appendix A.3. □

Part (ii) describes the estimated current market. Its equilibrium combines rate optimality, the first equation in (6), with value consistency, the second equation.

4 Equilibrium Structure

This section establishes an order structure of the equilibrium set (Section 4.1). From this structure, it derives a test of uniqueness that can be run in the estimated market and in the reported policy designs (Section 4.2). The section closes by summarizing what these results deliver for the empirical analysis.

4.1 Order Structure

An exposure regime specifies, for each doctor–post pair and route, whether the exposure rate is held fixed or follows the behavioral rule (2).

Definition 2 (Exposure regime)**.** *An exposure regime is a collection of exposure-rate functions* $\lambda = (\lambda_{ij}^c)_{i \in I,\, j \in J,\, c \in \mathcal{C}}$*, each mapping continuation values* α *to a nonnegative rate, in which every* λ_{ij}^c *is of one of two kinds:*

(a) (Behavioral) $\lambda_{ij}^c(\alpha) = \bar{\lambda}^c \exp\{s_{ij}^c - \omega_{ij}^c(\alpha)\}$*, as in equation* (2)*.*

(b) (Fixed) $\lambda_{ij}^c(\alpha)$ *is a nonnegative constant that does not depend on* α*.*

An equilibrium of the regime is a vector $\alpha \geq 0$ that solves $\alpha = F(\alpha)$, where $F(\alpha) = \Phi_K(\alpha; \lambda(\alpha))$ is the map in (5) *evaluated at the regime's exposure rates.*

An equilibrium α of a regime determines the exposure rates $\lambda(\alpha)$ and the threshold rules of Section 3.1. The two cases of Definition 1 are the regimes in which every rate is fixed and in which every rate is behavioral. In the estimated current market both routes are behavioral. The search-retained class of Section 6.1 fixes the agency rates at A_{ij}/K_i and keeps doctor search behavioral, and the full-replacement class fixes both routes, with the self route switched off. Write $F = (F^D, F^P)$ for the doctor-side and post-side components of the map. All results in this section hold in every exposure regime.

Two properties of F drive the analysis (Lemma 1 in Appendix A.4). First, the doctor-side values enter doctor i's equation only through α_i^D, and the post-side values enter post j's equation only through α_j^P. Second, every component of F is nonincreasing in every continuation value. A higher continuation value makes an agent more selective after an exposure. When the agent initiates a route behaviorally, a higher value also lowers the agent's exposure rate on that route. For the other side, both responses reduce the expected gain from an exposure to that agent. Hence, for given post values α^P, the doctor equations have a unique solution $\hat{\alpha}^D(\alpha^P)$, which is nonincreasing in α^P, and the post equations similarly define a nonincreasing $\hat{\alpha}^P(\alpha^D)$. Equilibria correspond one to one to the fixed points of the nondecreasing map

$$T(\alpha^P) = \hat{\alpha}^P(\hat{\alpha}^D(\alpha^P)),$$

with $\alpha^D = \hat{\alpha}^D(\alpha^P)$. Let $\bar{F}^P$ denote the post-side map evaluated at zero continuation values, which is the upper corner of the box used in the existence proof.

Proposition 2 (Equilibrium structure)**.** *In every exposure regime, the following hold.*

(i) Ordered by their post-side values, the equilibria form a nonempty complete lattice. For any two equilibria, $\alpha^D \geq \alpha'^D$ if and only if $\alpha^P \leq \alpha'^P$.

(ii) A doctor-optimal equilibrium $(\bar{\alpha}^D, \underline{\alpha}^P)$ and a post-optimal equilibrium $(\underline{\alpha}^D, \bar{\alpha}^P)$ exist, and every equilibrium satisfies $\underline{\alpha}^D \leq \alpha^D \leq \bar{\alpha}^D$ and $\underline{\alpha}^P \leq \alpha^P \leq \bar{\alpha}^P$.

(iii) The sequence $T^k(0)$ is nondecreasing and converges to $\underline{\alpha}^P$, and the sequence $T^k(\bar{F}^P)$ is nonincreasing and converges to $\bar{\alpha}^P$. For every k and every equilibrium, $T^k(0) \leq \alpha^P \leq T^k(\bar{F}^P)$. The equilibrium is unique if and only if the two limits coincide.

The doctor-optimal equilibrium is simultaneously the best equilibrium for every doctor slot and the worst for every post. Across equilibria, the interests of agents on the same side are aligned and those of agents on opposite sides are opposed. The proposition follows from Tarski's fixed-point theorem (Tarski, 1955) once the two properties above are established.

The same monotonicity signs the effects of parameters that shift only one side's equations (Milgrom and Roberts, 1994). A higher abandonment hazard for a doctor, or more slots for a doctor, weakly raises the values of posts and weakly lowers the values of doctors at both extremal equilibria. A higher withdrawal hazard for a post does the opposite (Proposition 4 in Appendix A.4). Recommendation exposure is not a parameter of this kind. A recommendation adds exposure flow for both sides, so the theory does not determine which side gains when the platform changes its recommendations. Section 6 answers this question quantitatively.

Relation to Adachi (2003). The model of Section 3 builds on the marriage-market search model of Adachi (2003), so it is worth stating what that paper establishes. In Adachi's model, men and women of different types search in discrete time and meet partners of the other side at random, according to a fixed distribution. Utility is nontransferable and determined by the two partners' types, preferences are strict, and a pair matches if both accept. Matched agents are replaced by identical agents, so the stocks of searchers do not change. Adachi characterizes stationary equilibria by the agents' reservation utilities, which correspond to the continuation values here. Using Tarski's theorem, he shows that the set of equilibria is nonempty and forms a complete lattice when ordered by the men's reservation utilities, in analogy to the lattice of stable matchings. His main result concerns the limit in which search costs vanish. As the discount factor tends to one, the set of equilibrium matchings reduces to the set of stable matchings of the corresponding Gale–Shapley marriage problem.

Proposition 2 extends this order structure to a market with the features this application requires. These features are two exposure routes with route-specific payoffs, doctors with several slots, pair- and route-specific exposure rates in place of random meetings, and logistic payoff shocks at each exposure. The exposure rates are either chosen by the initiating side or fixed by the platform, so the result covers every exposure regime, including the current market. The step that is not immediate is endogenous exposure. With random meetings, an agent's reservation utility affects the other side only through acceptance. Here it also changes how often the agent initiates an exposure, and by the second property above both effects move the other side's value in the same direction.

4.2 Uniqueness in the Estimated Market and Policy Designs

Proposition 2 does not rule out several equilibria. For the estimated market, uniqueness is therefore established computationally. Proposition 2(iii) brackets every equilibrium between two monotone sequences, but computing T requires solving each side's equations, and a numerical error could move a computed iterate past an equilibrium. The following version of the iteration replaces each solution by a bound that can be checked directly.

Proposition 3 (Certified bracket)**.** *Let $\ell^0 = 0$ and $u^0 = \bar{F}^P$. Given ℓ^k, let a^k be any vector with $a_i^k \geq F_i^D(a_i^k, \ell^k)$ for every i, and let ℓ^{k+1} be any vector with $\ell_j^{k+1} \leq F_j^P(a^k, \ell_j^{k+1})$ for every j. Given u^k, let b^k be any vector with $b_i^k \leq F_i^D(b_i^k, u^k)$ for every i, and let u^{k+1} be any vector with $u_j^{k+1} \geq F_j^P(b^k, u_j^{k+1})$ for every j. Then every equilibrium satisfies $\ell^k \leq \alpha^P \leq u^k$ and $b^k \leq \alpha^D \leq a^k$ for all k.*

Each inequality in Proposition 3 concerns a single agent's equation and is checked by one evaluation of F. In the computation, I solve each side's equations by Newton's method and then move each solution outward, starting from a relative margin of 10^{-11}, until its inequality holds. I call this procedure the test of Proposition 3. When the lower and upper bounds that it returns coincide up to numerical precision, I refer to the equilibrium as unique.

Applied to the estimated market, to the current recommendations, and to the best-found long-run designs of Section 6, the test finds a unique equilibrium in every case. Each of these equilibria is also regular, that is, the Jacobian of the equilibrium system is nonsingular there (Table A.1 in Appendix A.4). These results bear on the rest of the paper in four ways. First, the estimates and the long-run values of the current recommendations and of the best-found designs are not conditional on an equilibrium selection, so the comparisons among them in Section 6 reflect the policies alone. Second, because the equilibrium at the estimate is unique and regular, it remains unique for nearby parameter values and varies differentiably with them (Corollary 1), which justifies the analytic standard errors in Section 5. Third, because recommendations add exposure flow for both sides, the model does not determine in advance which side gains from a redesign; the gains and losses reported in Section 6 are properties of the estimated market. Finally, the test can be applied to any other allocation the platform might consider. It takes a few dozen iterations. Each iteration solves the doctor-side equations given the post-side values and then the post-side equations given the doctor-side values, the same computations that are used to find an equilibrium.

5 Empirical Strategy and Results

5.1 Parameterization and Likelihood

The model leaves the gross exposure values s_{ij}^c and the deterministic post-exposure payoffs u_{ijc}^D and u_{jic}^P unspecified. To connect these primitives to the observed exposure counts and bilateral decisions, I parameterize them using doctor characteristics, post characteristics, and variables defined for each doctor–post pair. I then impose the equilibrium consistency condition from Section 3.2 when evaluating the likelihood.

Let $c \in \{\text{self}, \text{agency}\}$ denote doctor-initiated search and agent-initiated screening. Let Z_{ij}^D and Z_{ij}^P denote the covariate vectors for doctor–post pair (i, j) that enter the doctor-side

and post-side indices, respectively. Each collects three groups of observed covariates: doctor characteristics, post characteristics, and variables defined for the doctor–post pair. The third group includes distance, indicators for preference, schedule, specialty, and task compatibility, and the five predetermined doctor–facility history measures. The first element of each vector is one.

Gross exposure values and exposure rates. Each doctor–post pair has two route-specific exposure rates: one for doctor-initiated search and one for agent-initiated screening. These rates determine the expected number of exposures observed through each route. Because the reference rate $\bar{\lambda}^c$ in equation (2) is constant within a route, I absorb $\log \bar{\lambda}^c$ into a route-specific constant. I write the resulting empirical parameterization as

$$s_{ij}^{\text{self}} + \log \bar{\lambda}^{\text{self}} = (Z_{ij}^D)'\beta_{\text{self}},$$
$$s_{ij}^{\text{agency}} + \log \bar{\lambda}^{\text{agency}} = (Z_{ij}^P)'\beta_{\text{agency}}.$$

Substituting these expressions into the rule (2) gives

$$\log \lambda_{ij}^{\text{self}} = (Z_{ij}^D)'\beta_{\text{self}} - \alpha_i^D, \tag{7}$$
$$\log \lambda_{ij}^{\text{agency}} = (Z_{ij}^P)'\beta_{\text{agency}} - \alpha_j^P. \tag{8}$$

A doctor's continuation value reduces doctor-initiated search, while a post's continuation value reduces agent-initiated screening.

Post-exposure payoffs and acceptance. The same three groups of covariates enter the two post-exposure payoff indices, but their coefficients differ by side and from the coefficients in the exposure-rate equations. I parameterize the deterministic payoffs in equations (3)–(4) as

$$u_{ijc}^D = (Z_{ij}^D)'\beta_D + \chi_D \mathbf{1}\{c = \text{self}\},$$
$$u_{jic}^P = (Z_{ij}^P)'\beta_P + \chi_P \mathbf{1}\{c = \text{agency}\}.$$

Because payoff coefficients and logit scales are not separately identified, I normalize $\zeta^D = \zeta^P = 1$. The resulting acceptance probabilities are

$$\Pr(a_{ijc}^D = 1) = \Lambda\Big((Z_{ij}^D)'\beta_D - \alpha_i^D + \chi_D \mathbf{1}\{c = \text{self}\}\Big),$$
$$\Pr(a_{jic}^P = 1) = \Lambda\Big((Z_{ij}^P)'\beta_P - \alpha_j^P + \chi_P \mathbf{1}\{c = \text{agency}\}\Big).$$

The two χ coefficients are route-specific differences in acceptance intercepts. For doctors, agent-initiated screening is the reference route, so χ_D is the change in the doctor acceptance index when the doctor initiates the exposure. For posts, doctor-initiated search is the reference route, so χ_P is the change in the post acceptance index when a platform agent initiates the exposure.

Observed likelihood and equilibrium restriction. The likelihood is defined over the doctor–post pairs constructed in Section 2.2. For each selected post episode in the monthly market, the observed route-specific exposure count satisfies

$$n_{ij}^c \sim \text{Poisson}\Big(K_i T_j \lambda_{ij}^c\Big),$$

where λ_{ij}^c is the monthly exposure rate for one doctor slot and T_j is the post episode's observed duration in months. Doctor i has K_i such slots, so $K_i T_j \lambda_{ij}^c$ is the expected number of route-c exposures of the pair over the post episode. The predetermined activity proxy fixes K_i before estimation and it enters with no estimated coefficient.

Write $\mathcal{C} = \{\text{self}, \text{agency}\}$ and let $\mathcal{M}$ denote the set of doctor–post pairs in the estimation market. For route c, let $\mathcal{A}_c^D$ and $\mathcal{A}_c^P$ denote the subsets with an observed doctor- and post-side acceptance decision, respectively. Set $m_{ij}^c = K_i T_j \lambda_{ij}^c$, $P_{ijc}^D = \Pr(a_{ijc}^D = 1)$, and $P_{jic}^P = \Pr(a_{jic}^P = 1)$. Suppressing the dependence of the rates and acceptance probabilities on (θ, α), the likelihood is

$$\begin{aligned} L(\theta, \alpha) = &\prod_{c \in \mathcal{C}} \prod_{(i,j) \in \mathcal{M}} \frac{\exp(-m_{ij}^c)(m_{ij}^c)^{n_{ij}^c}}{(n_{ij}^c)!} \\ &\times \prod_{c \in \mathcal{C}} \prod_{(i,j) \in \mathcal{A}_c^D} (P_{ijc}^D)^{a_{ijc}^D} (1 - P_{ijc}^D)^{1 - a_{ijc}^D} \\ &\times \prod_{c \in \mathcal{C}} \prod_{(i,j) \in \mathcal{A}_c^P} (P_{jic}^P)^{a_{jic}^P} (1 - P_{jic}^P)^{1 - a_{jic}^P}. \end{aligned} \tag{9}$$

Because $\mathcal{C}$ contains two routes, equation (9) has two Poisson count components and four route-by-side Bernoulli components. The route-specific constants inside λ_{ij}^c remain elements of θ and are estimated jointly rather than profiled out.

The continuation values α are not free parameters. At each θ, they must be equilibrium values. Let $\Phi(\theta, \alpha)$ denote the map Φ_K in (5) evaluated at the parameterized payoffs and at the exposure rates in (7)–(8). The condition $\alpha = \Phi(\theta, \alpha)$ is then the behavioral equilibrium system (6). The estimator maximizes the log likelihood $\ell(\theta, \alpha) = \log L(\theta, \alpha)$ subject to this restriction:

$$\max_{\theta, \alpha} \ \ell(\theta, \alpha) \quad \text{subject to} \quad \alpha = \Phi(\theta, \alpha).$$

5.2 Identification and Estimation Procedure

The exposure records and the decisions made after an exposure discipline different parts of the model jointly with the continuation values. Variation in doctor-initiated and agent-initiated exposure counts across pairs informs the two exposure-rate indices. Conditional on an exposure, variation in doctor and post decisions across pairs and routes informs the two acceptance indices and the two initiator-specific acceptance intercept differences.

A remaining identification concern is pair-specific information that doctors or platform agents observe but the researcher does not. For example, a doctor may apply to a hospital,

or an agent may recommend the hospital's post to that doctor, because the doctor has worked successfully with the facility before. The same familiarity can also make both sides more likely to accept after the exposure. If it is omitted, the estimated coefficients on distance, recorded match indicators, or the identity of the initiating side may partly capture relationship-driven exposure and acceptance. I therefore include the five predetermined doctor–facility history measures in every exposure-rate and acceptance index. These measures record prior exposures, prior contracting, recency, the route used in earlier exposures, and the absence of any recorded prior relationship. They place the most direct observable traces of persistent doctor–facility familiarity in both stages of the model.

The monthly doctor-abandonment hazard, post-withdrawal hazard, and discount rate are calibrated to 0.05, 0.10, and $-\log(0.99)$, respectively. The constrained maximization problem of Section 5.1 is a mathematical program with equilibrium constraints (MPEC). Appendix Section B.3 describes the MPEC problem, solver implementation, and convergence diagnostics.

I report standard errors clustered by doctor and by post, allowing dependence among doctor–post pairs that share a doctor or a post. The calculation also accounts for the dependence of continuation values on the structural parameters through the equilibrium conditions.[5] When differentiating the likelihood, I therefore include both the direct effects of parameter changes and their indirect effects through changes in continuation values. Appendix Section B.4 describes the analytic covariance calculation.

5.3 Estimates and Channel Fit

The fitted model preserves the sharp reversal in acceptance across routes and closely reproduces doctor acceptance after sorting. It predicts 2,521.52 doctor-initiated exposures, compared with 2,503 observed, and 955.70 agent-initiated exposures, compared with 1,013 observed. Doctor acceptance is 97.98 percent in the data and 97.97 percent in the model after sorting, and 28.35 versus 24.85 percent after screening. Post acceptance is 40.98 versus 41.97 percent after sorting and 98.43 versus 94.76 percent after screening. Thus, the model reproduces the route-side ordering while leaving some differences in acceptance levels, especially after screening.

Table 2 shows how the estimated associations with observed characteristics differ between exposure and acceptance. Entries are average probability changes, in percentage points, for a one-standard-deviation increase in a continuous variable or a change from zero to one in an indicator. Other covariates and continuation values are held fixed. Exposure entries average changes in the probability of at least one exposure during the observed post episode over doctor–post pairs. Acceptance entries average over observed decisions for the indicated route and

[5]The test of Proposition 3 shows that the equilibrium at the estimate is unique: it brackets every continuation value in an interval narrower than 2.2×10^{-9} (Table A.1 in Appendix A.4). The equilibrium is also regular, so by Corollary 1 the continuation values are a differentiable function of the parameters near the estimate.

Table 2. Selected Changes in Exposure and Acceptance Probabilities

	Exposure		Doctor acceptance		Post acceptance	
	Self	Agency	Self	Agency	Self	Agency
Log distance	-0.030	-0.006	0.176	1.494	0.129	0.026
	(0.003)	(0.002)	(0.215)	(1.934)	(1.350)	(0.275)
Hours	0.003	0.018	-0.831	-5.186	1.854	0.366
	(0.006)	(0.014)	(0.488)	(2.397)	(7.329)	(1.397)
Log posted salary	-0.010	-0.029	-0.167	-1.262	-31.159	-16.465
	(0.004)	(0.002)	(0.353)	(2.518)	(4.394)	(6.414)
Preference match	0.022	0.006	0.271	2.186	-0.899	-0.185
	(0.009)	(0.005)	(0.453)	(4.001)	(2.909)	(0.608)
Day match	0.097	0.010	1.504	12.438	4.440	0.888
	(0.020)	(0.005)	(0.302)	(4.202)	(2.722)	(0.500)
Specialty match	0.068	0.039	-0.765	-5.476	7.006	1.370
	(0.011)	(0.011)	(0.672)	(3.787)	(3.862)	(0.664)
Prior exposures†	0.059	0.037	0.089	0.782	-0.347	-0.078
(past 365 days)	(0.013)	(0.006)	(0.036)	(0.324)	(0.250)	(0.057)
Prior contract†	1.125	1.598	-1.379	-17.629	28.799	9.128
	(0.198)	(0.422)	(1.246)	(8.565)	(6.216)	(1.318)

Notes: Entries are probability changes in percentage points. Continuous covariates increase by one full-estimation-sample standard deviation; indicators change from zero to one. Self and Agency denote doctor-initiated and agent-initiated exposures. Exposure entries average changes in the probability of at least one exposure over the post episode across all retained doctor–post pairs, using their observed stock and duration offsets. Acceptance entries average over the original observed decision trials for the indicated side and route. Continuation values and all other covariates and offsets remain fixed. †History comparisons restrict each averaging population to pairs with a positive prior exposure count. Approximate standard errors in parentheses transform each coefficient's doctor–post clustered standard error by the delta method, holding the starting prediction indices fixed; they omit uncertainty in those starting predictions. Appendix B.4 gives the calculation.

side. The six columns distinguish the two exposure routes and each side's acceptance after either route. Shared acceptance coefficients can imply different probability changes because the starting probabilities and decision samples differ across routes. Parentheses contain approximate transformed standard errors; Appendix Section B.4 explains the calculation, and Table B.4 reports the underlying coefficients and clustered standard errors.

Greater distance is associated primarily with a lower probability of exposure. A one-standard-deviation increase in log distance lowers this probability by 0.030 percentage points through doctor-initiated search and 0.006 points through agent-initiated screening, relative to fitted averages of 0.090 and 0.033 percent. The acceptance changes are imprecisely estimated on both sides. Geographic proximity therefore predicts which pairs are exposed to each other more clearly than it predicts acceptance once an exposure occurs.

Job characteristics have different associations with the two sides' acceptance decisions. A one-standard-deviation increase in hours lowers doctor acceptance by 0.83 percentage points after search and 5.19 points after screening. Higher posted salary is associated mainly with lower screening and post acceptance: a one-standard-deviation increase in log salary lowers the screening exposure probability by 0.029 points and post acceptance by 31.16 and 16.47 points after search and screening, respectively. Its associations with doctor acceptance are small and imprecisely estimated.

Compatibility with a doctor's preferred work schedule predicts both exposure and doctor acceptance. A schedule match raises the two exposure probabilities by 0.097 and 0.010 percentage points, and doctor acceptance by 1.50 points after search and 12.44 points after screening. The difference between the doctor-side acceptance changes partly reflects the already high acceptance probability after doctor-initiated search. The post-side point estimates are also positive, at 4.44 and 0.89 points. Specialty matches likewise predict more exposure, but their acceptance associations differ by side: the point estimates are positive for posts and negative for doctors. Preferred-location matches predict more exposure, with less precisely estimated associations with acceptance. Compatibility measures thus have more consistent positive associations with exposure than with both sides' subsequent acceptance.

The relationship-history rows focus on pairs with at least one exposure in the preceding 365 days. Within this group, more prior exposures predict a higher probability of exposure through both routes and slightly higher doctor acceptance, with little change in post acceptance. A prior contract has a stronger association with post acceptance: the probability changes are 28.80 points after search and 9.13 points after screening. It also raises the two exposure probabilities by 1.125 and 1.598 points. Its doctor-side point estimates are negative, although the underlying coefficient's 95 percent interval includes zero. The frequency of prior exposures and prior contracting therefore capture different aspects of the relationship history associated with current exposure and acceptance.

6 Policy Exercises

The policy exercises change how the platform recommends jobs to doctors. They are motivated by the platform's consideration of an algorithmic recommendation system and ask three broad questions. First, should algorithmic recommendations operate alongside doctors' own search and applications, or should they replace doctor search entirely? Second, what should the algorithm prioritize: generating greater immediate expected surplus or creating greater long-run value after doctors and posts adjust? Third, is the choice of who receives recommendations as important as volume, that is, as increasing the number of recommendations or the size of the market?

Section 6.1 first explains how counterfactual recommendation policies are constructed. The next three subsections answer the three questions in the same order. Section 6.2 compares keeping doctor search with replacing it and shows why the model favors retaining decentralized search. Section 6.3 then conditions on doctor search remaining available and compares what the algorithm should prioritize. Finally, Section 6.4 keeps doctor search available and asks what the platform gains from making more recommendations, separates that change from reallocating recommendations across doctors and posts, and compares the gains from redesign with what could be achieved by expanding the market itself.

6.1 Policy Classes and Objectives

All counterfactual policies choose the same object, which I call the *policy matrix*:

$$A = (A_{ij})_{i \in I, j \in J}, \qquad A_{ij} \geq 0,$$

where A_{ij} is the expected number of monthly exposures that the platform assigns to doctor–post pair (i, j) through recommendations. Equivalently, A_{ij}/K_i is the recommendation rate for one slot of doctor i. A policy class determines which policy matrices are feasible and whether doctor search remains available. A policy objective determines how the platform chooses A within that class. I introduce these two ingredients in turn and then describe how the resulting policies are computed and compared.

6.1.1 Policy Class

For each policy class, the comparisons at current exposure budgets keep the expected number of exposures that the platform assigns to each doctor at the level estimated for the current market and change only which posts receive those exposures. This is because a policy that sharply changes this number for an individual doctor would be difficult to justify in practice.

I use fitted current exposures to define pair-specific upper bounds on the policy matrix. Let

$$\widehat{M}_{ij}^{\text{self}} = K_i \lambda_{ij}^{\text{self}}(\widehat{\alpha}_i^D) \qquad \text{and} \qquad \widehat{M}_{ij}^{\text{agency}} = K_i \lambda_{ij}^{\text{agency}}(\widehat{\alpha}_j^P)$$

denote the fitted monthly numbers of self-initiated and agency-initiated exposures under the current policy. Over a period of one month, define the pair-specific reference level

$$\overline{M}_{ij} = \max\{K_i, \widehat{M}_{ij}^{\text{self}} + \widehat{M}_{ij}^{\text{agency}}\}.$$

This reference allows one exposure per doctor slot for pair (i, j) unless the fitted current total is larger, so it never excludes the fitted current allocation. I define the portion that remains available for platform recommendations after accounting for fitted self-initiated exposures as

$$\overline{A}_{ij} = \max\{\overline{M}_{ij} - \widehat{M}_{ij}^{\text{self}}, 0\}.$$

The two policy classes use these quantities differently: $\overline{A}_{ij}$ caps recommendations when doctor search remains available, whereas $\overline{M}_{ij}$ caps recommendations when all exposures are assigned by the platform.

Search-retained class. The *search-retained* class changes platform recommendations while leaving doctor-initiated search governed by the estimated behavioral rule. Let b_i denote the expected monthly number of exposures that the platform assigns to doctor i through recommendations. For a feasible vector $b = (b_i)_{i \in I}$, the class has feasible set

$$\mathcal{B}^{\text{retain}}(b) = \left\{ A \geq 0 : \sum_j A_{ij} = b_i \ \forall i, \quad A_{ij} \leq \overline{A}_{ij} \right\}. \tag{10}$$

The baseline exercise uses each doctor's fitted current recommendation budget, $b_i^{\text{agency}} = \sum_j \widehat{M}_{ij}^{\text{agency}}$.

For a chosen A, the platform replaces the behavioral rule on the agency route with the forced rate A_{ij}/K_i. Doctor search continues to follow the behavioral rule (2), so continuation values and acceptance solve

$$(r + \delta_i^a)\alpha_i^D = \sum_j \left[\lambda_{ij}^{\text{self}}(\alpha_i^D)\psi_{ij,\text{self}}^D(\alpha) + \frac{A_{ij}}{K_i}\psi_{ij,\text{agency}}^D(\alpha) \right],$$
$$(r + w_j)\alpha_j^P = \sum_i \left[K_i \lambda_{ij}^{\text{self}}(\alpha_i^D)\psi_{ij,\text{self}}^P(\alpha) + A_{ij}\psi_{ij,\text{agency}}^P(\alpha) \right].$$

Write $\alpha^{\text{retain}}(A; K)$ for the solution. Thus A is the only object assigned directly by the platform; doctor search, acceptance, and continuation values respond in equilibrium.

Full-replacement class. The *full-replacement* class removes doctor-initiated search and assigns all exposures through platform recommendations. Its feasible set is

$$\mathcal{B}^{\text{replace}}(b) = \left\{ A \geq 0 : \sum_j A_{ij} = b_i \ \forall i, \quad A_{ij} \leq \overline{M}_{ij} \right\}.$$

The baseline exercise uses each doctor's fitted current total-exposure budget, $b_i^{\text{total}} = \sum_j (\widehat{M}_{ij}^{\text{self}} + \widehat{M}_{ij}^{\text{agency}})$. Under a candidate matrix A, no exposure arises on the self route and pair (i, j) receives A_{ij} exposures on the agency route, so continuation values solve

$$(r + \delta_i^a)\alpha_i^D = \sum_j \frac{A_{ij}}{K_i} \psi_{ij,\text{agency}}^D(\alpha),$$
$$(r + w_j)\alpha_j^P = \sum_i A_{ij} \psi_{ij,\text{agency}}^P(\alpha).$$

Write the solution as $\alpha^{\text{replace}}(A; K)$.

6.1.2 Policy Objective and Measurements

For a fixed policy class $k \in \{\text{retain}, \text{replace}\}$ and feasible b, I compare two objectives for choosing $A \in \mathcal{B}^k(b)$: immediate expected surplus and long-run value.

Short run: immediate expected surplus. The short-run problem allocates recommendations to maximize immediate expected bilateral surplus at the fitted continuation values. To state this problem, let the route-specific exposure matrices realized at continuation vector α be

$$M_{ij}^{c,k}(A; \alpha) = \begin{cases} K_i \lambda_{ij}^{\text{self}}(\alpha_i^D), & k = \text{retain},\ c = \text{self}, \\ A_{ij}, & k = \text{retain},\ c = \text{agency}, \\ 0, & k = \text{replace},\ c = \text{self}, \\ A_{ij}, & k = \text{replace},\ c = \text{agency}. \end{cases}$$

The objective is

$$S^k(A) = \sum_{i,j,c} M_{ij}^{c,k}(A; \widehat{\alpha}) \left\{ \psi_{ijc}^D(\widehat{\alpha}) + \psi_{ijc}^P(\widehat{\alpha}) \right\}.$$

The short-run rule maximizes $S^k(A)$ over $A \in \mathcal{B}^k(b)$. It holds opportunity costs and acceptance fixed at $\widehat{\alpha}$. In the search-retained class it also holds doctor-initiated exposures at $\widehat{M}^{\text{self}}$, so only the recommendation matrix changes immediately.

Long run: value after equilibrium adjustment. The long-run rule accounts for how a recommendation allocation changes the equilibrium. For each candidate A, I re-solve acceptance and continuation values. In the search-retained class, this calculation also updates the exposure rates of doctor search, $\lambda^{\text{self}}(\alpha^D)$. I then add the resulting continuation values across all doctor slots and posts. I refer to this sum as *total user value* in the model:

$$W^k(A; K) = \sum_i K_i \alpha_i^{D,k}(A; K) + \sum_j \alpha_j^{P,k}(A; K).$$

The first term counts one doctor-side continuation value for each of doctor i's K_i slots, and the second counts one post-side continuation value for each post. I call $W^k(A; K)$ the long-run

value of allocation A. The long-run problem maximizes $W^k(A;K)$ over $A \in \mathcal{B}^k(b)$. I report the best allocation found by a numerical search that satisfies a first-order stopping criterion.[6] The test of Proposition 3 shows that the equilibrium is unique at each best-found allocation and at the current recommendations (Table A.1 in Appendix A.4). The long-run comparisons below therefore do not depend on which equilibrium is selected.

Benchmark. I also define an even-allocation benchmark for measuring post-side access. The benchmark spreads each doctor's recommendations as evenly as the pair-specific limits allow. Let $\overline{C}^{\text{retain}} = \overline{A}$ and $\overline{C}^{\text{replace}} = \overline{M}$. The benchmark allocation is

$$R^k_{ij}(b) = \min\{\overline{C}^k_{ij}, \tau^k_i(b)\}, \qquad \sum_j R^k_{ij}(b) = b_i,$$

where $\tau^k_i(b)$ is chosen so that doctor i receives exactly b_i recommendations. For the baseline comparison, write $R^{\text{retain}} = R^{\text{retain}}(b^{\text{agency}})$ and $R^{\text{replace}} = R^{\text{replace}}(b^{\text{total}})$.

Comparison across policies. I compare the policies along two dimensions: how evenly they distribute post-side outcomes and how much value they generate according to the two objective functions. Because posts are the institutions that pay for the platform's service, the platform must also avoid recommendation policies that distribute the benefits of that service too unevenly across them; I use the post-side access index to capture this practical equity concern.

To compare the distribution of outcomes across posts, I define a post-specific outcome for each objective. In the short-run comparison, $y^{SR,k}_j(A)$ is post j's immediate expected surplus flow at the fitted continuation values. In the long-run comparison, $y^{LR,k}_j(A)$ is post j's equilibrium continuation value:

$$y^{SR,k}_j(A) = \sum_{i,c} M^{c,k}_{ij}(A;\widehat{\alpha})\psi^P_{ijc}(\widehat{\alpha}), \qquad y^{LR,k}_j(A) = \alpha^{P,k}_j(A;K).$$

For each post, I compare its outcome under A with its outcome under the corresponding even-allocation benchmark. For $q \in \{SR, LR\}$, the resulting post-side access index is

$$\mathcal{A}^{q,k}(A) = 100\left[1 - \text{Gini}_{j\in J}\left\{\frac{y^{q,k}_j(A)}{y^{q,k}_j(R^k)}\right\}\right],$$

[6] The search uses the gradient of total user value obtained by differentiating the equilibrium system, including the response of doctor search when it remains available. Each accepted update must raise the value computed after re-solving the equilibrium and satisfy the doctor-level budgets and pair-specific bounds. Each reported long-run allocation attains a relative first-order policy gap no greater than 10^{-6} and a maximum equilibrium residual no greater than 10^{-10}. Appendix C defines the gap and reports independent checks of the final allocations. These checks establish approximate first-order stationarity, not global optimality, so I retain the term "best-found" long-run rule. The remaining comparison allocations—the short-run rules, the even-allocation benchmarks, the intermediate steps of the volume exercise, and the market-expansion benchmarks—are solved from the fitted continuation values and have not been certified individually.

provided $y_j^{q,k}(R^k) > 0$. The benchmark scores 100 because, when $A = R^k$, every ratio inside the Gini coefficient equals one and the Gini coefficient of this constant vector is zero. A lower score means that posts' outcomes change by more unequal proportions relative to the benchmark.

6.2 Should Doctor Search Remain Available?

This subsection answers the first question: should doctor search remain available alongside algorithmic recommendations? The simulations show that retaining doctor search makes recommendation design less sensitive to which criterion the platform prioritizes.

Table 3 reports the comparison. Within each policy class, I report the short-run maximizing rule and the best-found long-run rule, together with the *current hybrid* as a comparison. The current hybrid is the estimated current market, in which doctor search operates alongside the platform's existing recommendations. I also evaluate each displayed allocation under both objective functions. Within each policy class, the short-run objective is reported as a percentage of the value attained by the short-run maximizing rule, so that rule equals 100. The long-run objective is reported as a percentage of the value attained by the best-found long-run rule, so that rule also equals 100. In the full-replacement panel, the current hybrid is an external two-route comparator rather than an element of $\mathcal{B}^{\text{replace}}(b)$. I evaluate its fitted outcomes using the same full-replacement benchmarks. Its normalized objective values can therefore exceed 100: in that panel, 100 is the value attained within the full-replacement policy class, not an upper bound for the external current hybrid.

As shown in Table 3, across the two counterfactual rules, the short- and long-run objective values and access indices all vary less when doctor search is retained, using the within-class benchmarks defined in Section 6.1. The platform can therefore prioritize immediate surplus or long-run value without the extreme deterioration in other dimensions in contrast to the case of full replacement. For example, when all exposures are assigned by the platform, the short-run rule attains only 42.20 percent of the best-found long-run value and a long-run access score of 12.03.

The differences are particularly small in the long run, consistent with doctor search providing an adjustment margin as recommendations change. With doctor search retained, the two rules produce long-run values between 98.64 and 100 percent of the best-found value and long-run access scores between 92.53 and 94.48. Doctors adjust how intensively they search as continuation values change: although the platform assigns the same 1,138.2 expected monthly recommendations under each rule, equilibrium doctor-initiated exposures range from 2,242.2 to 2,876.8. These responses are absent under full replacement, where all exposures are assigned by the platform.

Table 3. Access, Value, and Realized Exposure under Alternative Policies

Policy rule	Agency	Realized self	Realized total	Short access	Long access	Short objective	Long value
Panel A: Doctor search retained; agency screening redesigned							
Current hybrid	1,138.2	2,773.4	3,911.6	82.11	97.83	80.19	98.89
Short-run best found	1,138.2	2,242.2	3,380.5	43.37	94.48	100.00	98.64
Long-run best found	1,138.2	2,876.8	4,015.1	64.60	92.53	93.32	100.00
Panel B: Doctor search replaced; all exposures assigned by platform							
Current hybrid	1,138.2	2,773.4	3,911.6	72.17	92.95	63.23	100.88
Short-run best found	3,911.6	0.0	3,911.6	4.80	12.03	100.00	42.20
Long-run best found	3,911.6	0.0	3,911.6	76.24	97.14	71.05	100.00

Notes: Agency is the platform-controlled recommendation count. Realized self is the equilibrium doctor-initiated count, and realized total is their sum. The displayed Realized self and Realized total columns are the long-run equilibrium counts for each rule. In Panel A, doctor search is recomputed from continuation values under every screening matrix for the long-run evaluation; it is not held fixed. The short-run objective instead holds baseline self-search exposures and baseline continuation and surplus terms fixed. In Panel B, realized self exposures are zero for full-replacement candidates; the current hybrid is a comparison benchmark outside that candidate class. Access is the post-side index normalized to the even-allocation benchmark. The short objective and long value are normalized within policy class. Best-found is under maintained equilibrium selection and does not claim a global optimum. Agency budget and agency decision-pair caps are feasibility gates. Any realized-total overage created by endogenous doctor search is reported as a non-gating diagnostic.

6.3 What Should the Algorithm Prioritize?

Conditional on retaining doctor search, I argue that the platform should prioritize long-run value when designing its recommendations. Three considerations support this choice.

First, the current hybrid already performs close to the long-run benchmarks, whereas its short-run performance leaves more room for improvement. Panel A of Table 3 reports a long-run access score of 97.83 and long-run value equal to 98.89 percent of the best-found value. In contrast, its short-run access score is 82.11 and its short-run surplus is 80.19 percent of the maximum. Thus, both long-run measures are close to their benchmarks even though the short-run measures remain around 80. This pattern suggests that the platform already places implicit weight on long-run outcomes. That interpretation is consistent with my communications with the platform, in which its representatives emphasized that agents' work extends beyond raising short-term match success rates.

Second, maximizing short-run surplus can make both long-run outcomes worse than under the current hybrid. The short-run rule reduces long-run access from 97.83 to 94.48 and normalized long-run value from 98.89 to 98.64, so it is dominated by the current hybrid on these two measures. If recommendation design should work well beyond the criterion it directly maximizes, this is a reason to avoid focusing solely on immediate surplus.

Third, optimizing long-run value can improve welfare in both the short run and the long run

relative to the current hybrid, while accepting a reduction in access. The long-run rule raises the normalized short-run surplus objective from 80.19 to 93.32, a gain of 13.13 percentage points, while raising normalized long-run value from 98.89 to 100. Its short-run access score falls from 82.11 to 64.60, a reduction of about one fifth; long-run access also falls, from 97.83 to 92.53. The rule therefore improves welfare under both the short-run and the long-run criterion, although it requires a tradeoff with post-side access. Together with the current hybrid's strong long-run performance, these comparisons support prioritizing long-run value in recommendation design.

Uneven gains from redesign. Although the redesign raises total user value only modestly relative to the current hybrid, it produces uneven gains and losses across posts. Doctor-side value falls by 0.20 percent and post-side value rises by 6.78 percent, but only 45.85 percent of posts gain value. Grouping posts by their continuation values under the current hybrid, the lowest quintile gains 16.18 percent in aggregate, whereas the highest quintile loses 0.96 percent. The increase in post-side value is therefore not a uniform improvement in post-side outcomes. Nor is the shift of value from doctors to posts built into the model: recommendations add exposure flow for both sides, so the theory in Section 4.1 does not determine which side gains from reallocating them. The shift is a property of the estimated market.

How does the redesign change matches and recommendations? The search-retained redesign raises the total number of matches, although it changes the routes through which they occur. Relative to the current hybrid, recommendation-initiated matches rise from 203.4 to 355.6 per month, while doctor-initiated matches fall from 945.8 to 894.4. Total matches therefore rise from 1,149.2 to 1,250.0; they also exceed the 1,168.3 matches under the short-run rule. The gain in continuation value reflects this larger number of matches after allowing for the different gains obtained from each doctor–post pair and route. Appendix Table C.2 separates the match number, match composition, and conditional-gain contributions. The allocation of recommendations is therefore important both for the matches it creates directly and for the matches that continue to arise through doctor search.

Table 4 shows which recommendations produce these changes. It compares the current and best-found allocations with doctor search retained. Each doctor's recommendation total is fixed, so the differences show which posts receive the existing recommendations.

The best-found long-run rule shifts recommendations toward shorter jobs and more matches to doctors' preferred work schedules, while assigning more weight to distant posts and fewer recommendations to specialty matches. Recommendation-weighted job duration falls from 11.19 to 6.43 hours. Recommendation-weighted geometric-mean distance rises from 28.7 to 37.2 kilometers. The share of recommendations matching the doctor's preferred combination of weekday and shift type rises by 18.62 percentage points. Preferred-location matches rise slightly, by

Table 4. How the Best-Found Long-Run Rule Retargets Recommendations

Recommendation-weighted attribute	Current	Best found	Change
Job duration (hours)	11.19	6.43	−4.76
Mean log distance	3.356	3.617	+0.261
Preference match (%)	36.32	37.05	+0.73 pp
Day match (%)	35.73	54.35	+18.62 pp
Specialty match (%)	31.61	7.58	−24.03 pp

Notes: Entries weight observed doctor–post attributes by platform-controlled recommendations. Current refers to fitted agency recommendations, and Best found refers to the best-found long-run recommendation matrix when doctor search remains available. Each doctor's total number of recommendations is held at its fitted current level. Equilibrium doctor search is recomputed when evaluating the policy but is not used as a weight in this table.

0.73 percentage points, whereas specialty matches fall by 24.03 percentage points. Thus, the higher-value allocation does not improve every familiar measure of pair compatibility: it favors shorter jobs and preferred work schedules while accepting longer distances and fewer specialty matches.

This exercise changes which posts receive the existing recommendations. The remaining question is whether the platform can create larger gains by increasing the number of recommendations rather than only changing their allocation.

6.4 How Should the Platform Make More Recommendations?

I organize this exercise around three sequential policy changes: recommendation expansion, equalization across doctors, and pairwise redesign. These changes correspond to three operational decisions: how many recommendations the platform makes, whether doctors receive equal recommendation opportunities, and which doctor–post pairs receive those recommendations. First, the platform doubles the total number of recommendations. Second, holding that larger total fixed, it assigns the same expected number of recommendations to every doctor. Third, holding both the aggregate volume and each doctor's recommendation total fixed, it redesigns which posts receive those recommendations.

To construct the first step, I start from doctor i's fitted current recommendation total,

$$b_i^{\text{agency}} = \sum_j \widehat{M}_{ij}^{\text{agency}},$$

in the feasible set in equation (10). The doubled profile assigns doctor i a recommendation total of $2b_i^{\text{agency}}$. For the second step, I keep the same doubled aggregate volume but assign every doctor the same expected number of recommendations:

$$b_i^{\text{eq}} = \frac{2\sum_\ell b_\ell^{\text{agency}}}{|I|}.$$

The equalization step applies only to recommendations assigned by the platform; endogenous doctor search can still make total exposures differ across doctors. For the third step, I hold b^{eq} fixed and search over

$$A \in \mathcal{B}^{\text{retain}}(b^{\text{eq}})$$

for the best-found long-run allocation of each doctor's recommendations across posts.

Panel A of Table 5 reports the percentage gains in total user value from these three changes. Doubling the current recommendation profile raises value by 0.65 percent. Equalizing recommendation totals across doctors adds a further 0.44 percent, and redesigning the allocation across posts at those equal totals adds 1.80 percent. Each step's percentage gain is measured relative to the policy immediately before it. The overall comparison in Panel A shows that the three changes together raise value by 2.91 percent relative to the current hybrid. Pairwise redesign contributes more than the first two steps combined, giving the platform a reason to improve the allocation of recommendations even when it can increase their number. Panel A also reports how each step divides its gains between doctors and posts, and Panel B reports how exposures and matches change in the final step. Appendices C.1 and C.2 discuss these results.

Market expansion versus recommendation redesign. To put the 1.80 percent value gain from pairwise redesign in perspective, I compare it with the gains from attracting additional doctors or posts. From the platform's perspective, recommendation redesign and market expansion are both ways to raise user value. I therefore ask how much targeted market expansion would generate the same increase in total user value.

The market-expansion benchmark starts from the doubled recommendation profile with equal totals across doctors, before pairwise redesign. It therefore compares market expansion with a 1.80 percent increase in value relative to that profile, matching the final step in Table 5. Doctor-side expansion increases the number of slots available from selected doctor types. On the post side, let L_j denote the number of identical copies of observed post j, with $L_j = 1$ in the observed market. Aggregate exposures directed to post type j are divided across its copies, so each copy receives route-c exposure flow M_{ij}^c/L_j. The corresponding long-run value is

$$W^{\text{retain}}(A; K, L) = \sum_i K_i \alpha_i^{D,\text{retain}}(A; K, L) + \sum_j L_j \alpha_j^{P,\text{retain}}(A; K, L).$$

I rank doctors by $\widehat{\alpha}_i^D$ and observed posts by $\widehat{\alpha}_j^P$, and add stock to the top decile on each side. Each addition is distributed across the selected types in proportion to their baseline stocks. During this exercise, I hold the platform recommendation matrix fixed at that pre-redesign profile and re-solve the equilibrium after each change in market stocks, allowing doctor search and continuation values to adjust.[7]

[7] I consider seven levels of doctor-slot additions and seven levels of post additions, and solve the equilibrium

Table 5. Recommendation Expansion, Equalization across Doctors, and Pairwise Redesign

Panel A: Value changes and who gains

	Value change (%)			Share with higher value (%)	
Policy step	Total	Doctors	Posts	Doctor slots	Posts
Double current recommendations	+0.65	−1.77	+10.97	2.17	100.00
Equalize recommendation totals	+0.44	−0.83	+5.22	16.25	84.04
Redesign pair allocation	+1.80	+1.60	+2.50	97.18	57.88
Overall: current to final	+2.91	−1.03	+19.68	17.47	70.44

Panel B: Exposures and matches in the final redesign step

	Change (%)		Conversion rate (%)	
Route	Exposures	Matches	Before	After
Doctor search	−25.40	+0.09	16.26	21.81
Recommendations	0.00	+29.94	24.42	31.73

Notes: Doctor search remains available throughout. The first three rows of Panel A compare each step with the preceding policy; Overall compares the final redesigned allocation with the current hybrid. Each percentage change uses the corresponding total or side-specific value under the baseline for that row. Share with higher value is the percentage of doctor slots or posts whose continuation value exceeds that baseline by more than 10^{-4}. Doctors are weighted by their numbers of slots; posts receive equal weight. Overall shares are computed directly from current and final values for each unit. Panel B compares doubled recommendations with equal doctor totals against the final redesigned allocation. Conversion is accepted matches divided by exposures on the indicated route. All entries condition on the estimated model and re-solved equilibria.

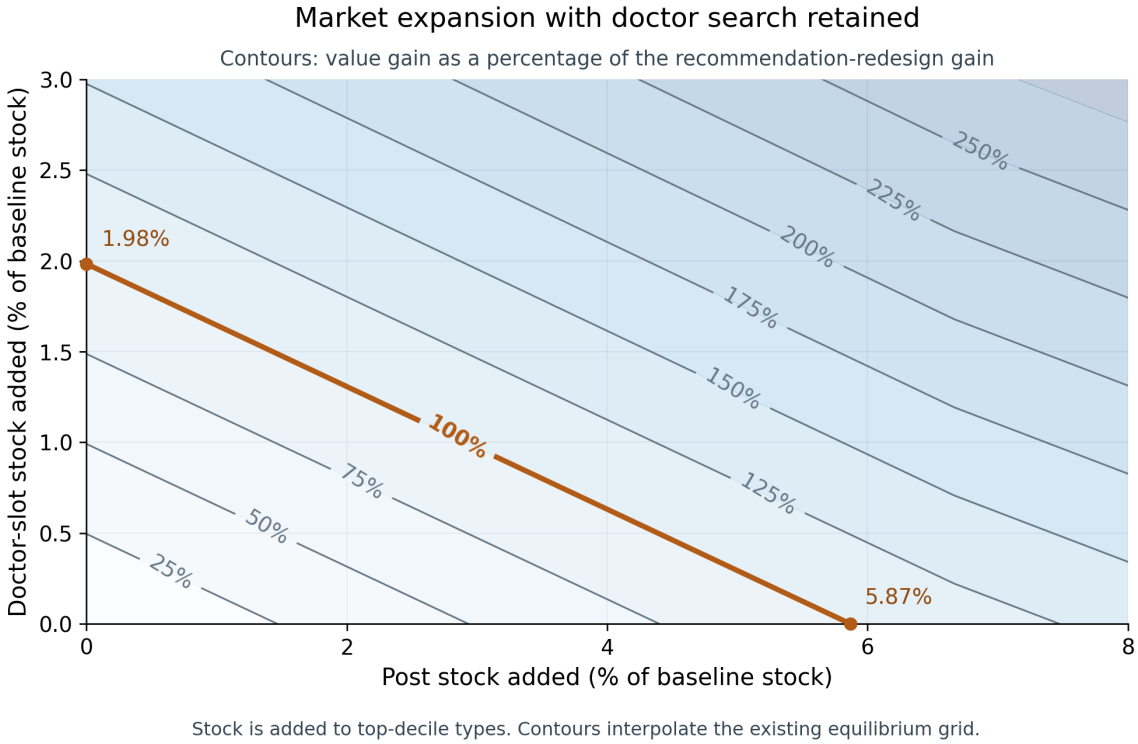

Figure 1. Market-composition equivalents of the equal-budget redesign increment with doctor search retained. Contours show the value gain as a percentage of the redesign increment; the highlighted 100 percent contour matches that increment. All contours interpolate the evaluated equilibrium grid.

With post stock fixed, linear interpolation between the evaluated grid points matches the redesign gain with additional slots from top-decile doctor types equal to 1.98 percent of aggregate baseline doctor-slot stock. With doctor stock fixed, the corresponding top-decile post addition is 5.87 percent of aggregate baseline post stock. Figure 1 shows the interpolated value contours, with the contour matching the redesign gain highlighted. These results put the scale of pairwise redesign in perspective. Improving the allocation of existing recommendations can substitute for some market expansion, although the preferred investment also depends on the costs of redesign and of attracting additional doctors or posts.

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for all 49 combinations. Each addition is expressed as a percentage of the corresponding side's total stock in the baseline market. For example, a 1 percent doctor-slot addition allocates new slots totaling 1 percent of all baseline doctor slots to the selected top-decile doctor types; it does not increase each selected type's slots by 1 percent. Post additions are defined analogously.

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A Model Details and Proofs

A.1 Route-Specific Closed Forms

For route c, write $\mu^D_{ijc} := u^D_{ijc}$ and $\mu^P_{jic} := u^P_{jic}$ for the deterministic payoffs parameterized in Section 5.1. If ε is logistic with scale ζ and location zero, then

$$\mathbb{E}[(\mu + \varepsilon - \alpha)_+] = \zeta \log\left(1 + \exp\{(\mu - \alpha)/\zeta\}\right).$$

Consequently,

$$\begin{aligned} q^D_{ijc}(\alpha) &= \zeta^D \log\left(1 + \exp\{(\mu^D_{ijc} - \alpha^D_i)/\zeta^D\}\right), \\ q^P_{ijc}(\alpha) &= \zeta^P \log\left(1 + \exp\{(\mu^P_{jic} - \alpha^P_j)/\zeta^P\}\right), \\ \psi^D_{ijc}(\alpha) &= \frac{q^D_{ijc}(\alpha)}{1 + \exp\{(\alpha^P_j - \mu^P_{jic})/\zeta^P\}}, \qquad \psi^P_{ijc}(\alpha) = \frac{q^P_{ijc}(\alpha)}{1 + \exp\{(\alpha^D_i - \mu^D_{ijc})/\zeta^D\}}. \end{aligned} \tag{11}$$

These expressions make clear why route rates cannot be collapsed once either initiator-specific acceptance intercept difference is nonzero. For example,

$$\lambda^{\text{self}}_{ij}\psi^D_{ij,\text{self}} + \lambda^{\text{agency}}_{ij}\psi^D_{ij,\text{agency}} \neq (\lambda^{\text{self}}_{ij} + \lambda^{\text{agency}}_{ij})\psi^D_{ij}$$

for any route-common ψ^D_{ij} in general. Setting both χ terms to zero recovers the nested route-common acceptance specification.

A.2 Auxiliary Representation of Behavioral Exposure Rates

The behavioral exposure equations admit the relative-entropy representation in equation (1). For a Poisson rate λ and baseline $\bar{\lambda} > 0$, the KL divergence rate defined in the main text is

$$D_{\text{KL}}(\lambda \| \bar{\lambda}) = \lambda \log(\lambda/\bar{\lambda}) - \lambda + \bar{\lambda}.$$

Its derivative with respect to λ is $\log(\lambda/\bar{\lambda})$. The auxiliary objective

$$\lambda(s - \omega) - D_{\text{KL}}(\lambda \| \bar{\lambda})$$

is strictly concave for $\lambda > 0$, and its unique first-order solution is

$$\log \lambda = \log \bar{\lambda} + s - \omega.$$

The adjustment cost enters this objective with unit weight on both exposure routes, which normalizes the units of s. Hence the relevant continuation value enters each behavioral log rate

with coefficient minus one, not with an estimated route-specific loading. For the doctor-search route, $\omega = \alpha_i^D$; for behavioral agent screening at the estimated current-market equilibrium, $\omega = \alpha_j^P$. A constant $\log \bar{\lambda}$ is observationally part of the route's exposure intercept, so the implementation does not estimate a separate baseline-rate parameter.

This representation supplies intuition for the log-rate specification, not a second payoff term in the stationary-renewal problem. In particular, the KL expression is absent from the continuation-value equations and from the counterfactual value objective. In the search-retained counterfactual the platform also replaces the behavioral agency first-order condition with a fixed policy matrix, while the doctor-search first-order condition continues to determine $\lambda^{\text{self}}(\alpha^D)$.

A.3 Proof of Proposition 1

Proof. Part (i), characterisation. Let (a, α) be a fixed-rate stationary-renewal equilibrium. Because the payoff shocks are continuous, ties occur with probability zero, and optimality requires the threshold rules in Section 3.1. Consider one waiting slot of doctor i over an interval of length dt. A route-c exposure to post j occurs with probability $\lambda_{ij}^c dt + o(dt)$. Relative to continued waiting, its expected value is

$$\mathbb{E}\Big[a_{jic}^P (U_{ij}^c - \alpha_i^D)_+\Big] = \psi_{ijc}^D(\alpha).$$

Discounting and abandonment reduce the waiting value at rate $r+\delta_i^a$. Stationary renewal leaves the environment after every event unchanged. Thus

$$\alpha_i^D = (1 - (r + \delta_i^a)dt)\alpha_i^D + \sum_{j,c} \lambda_{ij}^c dt\, \psi_{ijc}^D(\alpha) + o(dt),$$

which gives the doctor equation in (5). The post equation follows in the same way, except that the K_i independent slots generate a total route-c flow $K_i \lambda_{ij}^c$ from doctor i to post j. Hence every equilibrium continuation vector is a fixed point. Conversely, a fixed point together with the associated threshold rules satisfies optimal acceptance and value consistency, and therefore is a stationary-renewal equilibrium.

Part (i), existence. For fixed nonnegative finite route rates, every function ψ_{ijc}^D and ψ_{ijc}^P in (11) is continuous and nonnegative. It is also nonincreasing in every continuation-value argument on which it depends. Therefore $\Phi_K(\cdot; \lambda)$ is continuous and, for all $\alpha \geq 0$,

$$0 \leq \Phi_K(\alpha; \lambda) \leq \Phi_K(0; \lambda)$$

componentwise. Define the box

$$B = \prod_k [0, \Phi_{K,k}(0; \lambda)].$$

It is nonempty, compact, and convex, and $\Phi_K(B; \lambda) \subseteq B$. Brouwer's fixed-point theorem gives at least one fixed point in B.

Part (ii). By Definition 1, a triple (λ, a, α) is a behavioral stationary-renewal equilibrium if and only if $\lambda = \Gamma(\alpha; s)$ and (a, α) is a fixed-rate equilibrium at λ. The rates $\Gamma(\alpha; s)$ are nonnegative and finite, so part (i) applies. The second condition therefore holds if and only if $\alpha = \Phi_K(\alpha; \lambda)$ and a consists of the threshold rules generated by α, which gives the characterisation. For existence, substitute $\Gamma(\alpha; s)$ into Φ_K. The substitution preserves continuity and nonnegativity. Both Γ and the expected-gain terms are componentwise nonincreasing on the nonnegative orthant, so

$$0 \le \Phi_K(\alpha; \Gamma(\alpha; s)) \le \Phi_K(0; \Gamma(0; s))$$

componentwise. The box whose upper corner is the last expression is invariant, so Brouwer's theorem gives a solution of (6). □

The search-retained counterfactual is a hybrid instance of the existence argument in part (ii): the agency matrix is fixed, while $\lambda^{\text{self}}(\alpha^D)$ remains continuous, nonnegative, and bounded above by its value at zero. Only the full-replacement benchmark holds both route matrices fixed while continuation values and acceptance are re-solved.

A.4 Proofs for Section 4

This appendix proves Propositions 2 and 3 and the one-sided comparative statics of Section 4.1. It also reports two results that the main text does not use: a sufficient condition for uniqueness (Proposition 5) and an example with three equilibria (Remark 1).

Fix an exposure regime and write $F = (F^D, F^P)$ for the continuation-value map with the regime's exposure rates substituted. Call a pair and route with a positive exposure rate an edge $e = (i, j, c)$. Write $e \ni i$, $e \ni j$, and $e \in (i, j)$ for the edges at doctor i, at post j, and between them. Let $f_e^D = \lambda_{ij}^c \psi_{ijc}^D$ and $f_e^P = K_i \lambda_{ij}^c \psi_{ijc}^P$, so that $F_i^D = \sum_{e \ni i} f_e^D/(r + \delta_i^a)$ and $F_j^P = \sum_{e \ni j} f_e^P/(r + w_j)$. Let $b_e^D = 1$ if the rate of edge e follows the behavioral rule with the doctor as initiator and $b_e^D = 0$ otherwise, and define b_e^P analogously for posts.

Lemma 1 (Separability and monotonicity)**.** *(a) F_i^D depends on the doctor-side values only through α_i^D, and F_j^P depends on the post-side values only through α_j^P. (b) Every component of F is continuously differentiable and nonincreasing in every continuation value. For every edge,*

$$-\frac{\partial \log f_e^D}{\partial \alpha_i^D} = o_e^D := \frac{P_{ijc}^D}{q_{ijc}^D} + b_e^D, \qquad -\frac{\partial \log f_e^D}{\partial \alpha_j^P} = x_e^D := \frac{1 - P_{jic}^P}{\zeta^P} + b_e^P,$$

and the corresponding coefficients for f_e^P are $o_e^P = P_{jic}^P/q_{ijc}^P + b_e^P$ and $x_e^P = (1 - P_{ijc}^D)/\zeta^D + b_e^D$.

Proof. The factor $\psi_{ijc}^D = P_{jic}^P q_{ijc}^D$ depends on the doctor side only through q_{ijc}^D, a function of α_i^D, and on the post side only through P_{jic}^P, a function of α_j^P. A behavioral self rate depends only on α_i^D, and a behavioral agency rate only on α_j^P. This proves (a). By the closed forms in Appendix A.1, $\partial q_{ijc}^D/\partial \alpha_i^D = -P_{ijc}^D$ and $\partial \log P_{jic}^P/\partial \alpha_j^P = -(1 - P_{jic}^P)/\zeta^P$, and a behavioral rate

satisfies $\partial \log \lambda_{ij}^c / \partial \omega_{ij}^c = -1$ by (2). Adding the logarithmic derivatives of the factors gives the stated coefficients, which are nonnegative. The post side is symmetric. □

Proof of Proposition 2. Fix α^P and let $h_i(x) = x - F_i^D(x, \alpha^P)$, where, by Lemma 1(a), x stands for α_i^D. By Lemma 1(b), h_i is continuously differentiable with $h_i' \geq 1$, $h_i(0) \leq 0$, and $h_i(x) \geq x - F_i^D(0, \alpha^P)$ for $x \geq 0$. Hence h_i has a unique root $\hat{\alpha}_i^D(\alpha^P) \geq 0$. The root is continuous in α^P by the implicit function theorem and nonincreasing in α^P because F_i^D is. The same argument defines $\hat{\alpha}^P(\alpha^D)$. By Lemma 1(a), α solves $\alpha = F(\alpha)$ if and only if $\alpha^D = \hat{\alpha}^D(\alpha^P)$ and $\alpha^P = \hat{\alpha}^P(\alpha^D)$, that is, if and only if α^P is a fixed point of T and $\alpha^D = \hat{\alpha}^D(\alpha^P)$.

The map T is continuous and, as the composition of two nonincreasing maps, nondecreasing. Because both of its arguments are nonnegative, $T_j(\alpha^P) = F_j^P(\hat{\alpha}^D(\alpha^P), T_j(\alpha^P)) \leq F_j^P(0,0) = \bar{F}_j^P$. Hence T maps the complete lattice $[0, \bar{F}^P]$ into itself, and every fixed point lies in it. By Tarski's theorem, the fixed points form a nonempty complete lattice with a least element $\underline{\alpha}^P$ and a greatest element $\bar{\alpha}^P$. If $\alpha^P \leq \alpha'^P$, then $\alpha^D = \hat{\alpha}^D(\alpha^P) \geq \hat{\alpha}^D(\alpha'^P) = \alpha'^D$; if $\alpha^D \geq \alpha'^D$, then $\alpha^P = \hat{\alpha}^P(\alpha^D) \leq \hat{\alpha}^P(\alpha'^D) = \alpha'^P$. This proves (i), and (ii) follows with $\bar{\alpha}^D = \hat{\alpha}^D(\underline{\alpha}^P)$ and $\underline{\alpha}^D = \hat{\alpha}^D(\bar{\alpha}^P)$. For (iii), $0 \leq T(0)$ and monotonicity give $T^k(0) \leq T^{k+1}(0)$, and $0 \leq \alpha^P$ gives $T^k(0) \leq T^k(\alpha^P) = \alpha^P$ for every equilibrium. The sequence is bounded and nondecreasing, so it converges; its limit is a fixed point by continuity and lies below every equilibrium, so it equals $\underline{\alpha}^P$. The sequence from $\bar{F}^P$ is symmetric, and the equilibrium is unique if and only if $\underline{\alpha}^P = \bar{\alpha}^P$. □

Proposition 4 (One-sided shocks)**.** *Suppose that a parameter θ weakly raises F^P and weakly lowers F^D at every α. Then $\underline{\alpha}^P$ and $\bar{\alpha}^P$ are nondecreasing in θ, and $\underline{\alpha}^D$ and $\bar{\alpha}^D$ are nonincreasing. This applies to an increase in a doctor's abandonment hazard δ_i^a and to an increase in a doctor's number of slots K_i, holding fixed the per-slot behavioral rule and any recommendation counts A_{ij}. With the roles of the two sides exchanged, it applies to an increase in a post's withdrawal hazard w_j.*

Proof. For fixed α^P, a higher θ lowers $F_i^D(x, \alpha^P)$ and hence raises h_i, so $\hat{\alpha}^D(\alpha^P; \theta)$ is nonincreasing in θ; likewise $\hat{\alpha}^P(\alpha^D; \theta)$ is nondecreasing in θ. Because $\hat{\alpha}^P$ is nonincreasing in α^D, $T(\alpha^P; \theta)$ is nondecreasing in θ at every α^P. On a box that contains $[0, \bar{F}^P]$ for both parameter values, the least and greatest fixed points of a nondecreasing map are then nondecreasing in θ (Milgrom and Roberts, 1994). The doctor-side extremes $\hat{\alpha}^D(\underline{\alpha}^P; \theta)$ and $\hat{\alpha}^D(\bar{\alpha}^P; \theta)$ fall because both arguments move against doctors. In (5), δ_i^a enters only the denominator of F_i^D, and a larger K_i raises the post-side flows from doctor i while dividing any fixed recommendation count A_{ij} among more slots. □

Lemma 2 (Own effects dominate cross effects)**.** *For every edge, $x_e^P < o_e^D$ and $x_e^D < o_e^P$.*

Proof. Let $z = (u^D_{ijc} - \alpha^D_i)/\zeta^D$ and let Λ be the logistic distribution function, so that $P^D_{ijc} = \Lambda(z)$ and $q^D_{ijc} = \zeta^D \log(1+e^z)$. The terms b^D_e cancel, and $x^P_e < o^D_e$ is equivalent to $\{1-\Lambda(z)\}\log(1+e^z) < \Lambda(z)$, that is, to $\log(1+e^z) < e^z$, which holds for every z. Equivalently, the expected gain q^D_{ijc} is strictly log-concave in the payoff index. The second inequality is symmetric. □

Lemma 3 (Best-response derivatives)**.** *At an equilibrium, let* $t_e = f^D_e/(r+\delta^a_i)$, $v_e = f^P_e/(r+w_j)$, $D_i = 1 + \sum_{e\ni i} t_e o^D_e$, *and* $Q_j = 1 + \sum_{e\ni j} v_e o^P_e$. *Then*

$$\frac{\partial \hat\alpha^D_i}{\partial \alpha^P_j} = -E^D_{ij}, \quad E^D_{ij} = \frac{\sum_{e\in(i,j)} t_e x^D_e}{D_i}, \qquad \frac{\partial \hat\alpha^P_j}{\partial \alpha^D_i} = -E^P_{ji}, \quad E^P_{ji} = \frac{\sum_{e\in(i,j)} v_e x^P_e}{Q_j},$$

and the Jacobian of T *is* $DT = E^P E^D \geq 0$.

Proof. Apply the implicit function theorem to h_i, using Lemma 1(b), and the chain rule to T. □

Proposition 5 (Uniqueness without cycles)**.** *Let* G *be the bipartite multigraph whose vertices are the doctors and the posts and which has one edge for each pair and route with a positive exposure rate. If* G *has no cycle, which in particular requires that no pair be connected by both routes, the equilibrium is unique in every regime, and at the equilibrium*

$$\rho(DT) \leq \max_e \frac{x^D_e x^P_e}{o^D_e o^P_e} \cdot \max_i\Big(1 - \frac{1}{D_i}\Big) \cdot \max_j\Big(1 - \frac{1}{Q_j}\Big) < 1.$$

Proof. Let α be an equilibrium. Because G has no cycle, each pair is connected by at most one edge, so $E^D_{ij} = t_e x^D_e/D_i$ and $E^P_{ji} = v_e x^P_e/Q_j$ for the edge $e \in (i,j)$. Let M be the square matrix indexed by $I \cup J$ with blocks E^D and E^P off the diagonal and zeros on it. Since M^2 is block diagonal with blocks $E^D E^P$ and $E^P E^D$, which share their nonzero eigenvalues, $\rho(DT) = \rho(M)^2$. The positive entries of M lie on the edges of the forest G, in both directions. Root each tree, set $\Delta_v = 1$ at the root, and set $\Delta_v = \Delta_u (M_{uv}/M_{vu})^{1/2}$ for a vertex v with parent u. Then $\Delta M \Delta^{-1}$ has off-diagonal blocks S and $S^\top$ with $S_{ij} = (E^D_{ij} E^P_{ji})^{1/2}$, so $\rho(M)$ equals the largest singular value $\sigma_{\max}(S)$. With $a_e = t_e o^D_e/D_i$, $b_e = v_e o^P_e/Q_j$, and $\eta_e = x^D_e x^P_e/(o^D_e o^P_e)$, which is below one by Lemma 2, we have $S^2_{ij} = \eta_e a_e b_e$, $\sum_{e\ni i} a_e = 1 - 1/D_i$, and $\sum_{e\ni j} b_e = 1 - 1/Q_j$. For unit vectors y and z, the Cauchy–Schwarz inequality gives

$$y^\top S z \leq \Big(\max_e \eta_e\Big)^{1/2} \Big(\sum_i y_i^2 \sum_{e\ni i} a_e\Big)^{1/2} \Big(\sum_j z_j^2 \sum_{e\ni j} b_e\Big)^{1/2},$$

which yields the displayed bound and hence $\det(I - DT) > 0$ at every equilibrium. Finally, T is continuously differentiable on all of $\mathbb{R}^{|J|}$, because $\hat\alpha^D$ is defined and nonnegative for every α^P, and it maps $\mathbb{R}^{|J|}$ into $[0, \bar F^P]$. On the open box $U = \prod_j (-1, \bar F^P_j + 1)$, the homotopy $\alpha^P - sT(\alpha^P)$, $s \in [0,1]$, has no zero on the boundary, so the Brouwer degree of $I - T$ on U equals one. Because every fixed point of T is regular with $\det(I - DT) > 0$, the degree equals the number of fixed points, which is therefore one. □

Remark 1. *Without the no-cycle condition, multiple equilibria can occur in small markets, in line with the multiplicity that arises in two-sided search with nontransferable utility (Burdett and Wright, 1998). Consider two doctors and two posts with one route, fixed exposure rates of two per pair and month, $K_i = 1$, the calibrated hazards and discount rate, and unit logistic scales. Let $u^D_{11} = u^D_{22} = u^P_{12} = u^P_{21} = 12$ and set the other payoffs to zero, so that each doctor's favorite post prefers the other doctor. There are three equilibria. In the doctor-optimal equilibrium every doctor slot has continuation value 9.58, every post has 2.05, and almost all matches form in the doctors' favorite pairs. In the post-optimal equilibrium the values are 2.68 and 6.48, and almost all matches form in the posts' favorite pairs. The middle equilibrium has $\rho(DT) > 1$. The equilibrium is unique when the payoff 12 is replaced by 11. Analogous examples exist for a single doctor–post pair connected by both routes when the doctor's payoff is higher on the self route and the post's payoff is higher on the agency route.*

With two routes, a single doctor–post pair connected by both routes already forms a cycle, and the estimated exposure network has many cycles. Proposition 5 therefore does not apply to the estimated market, where uniqueness is verified with Proposition 3 instead.

Proof of Proposition 3. Let α be an equilibrium and suppose that $\ell^k \le \alpha^P$. Because $x - F^D_i(x, \ell^k)$ is increasing in x with root $\hat{\alpha}^D_i(\ell^k)$, the condition $a^k_i \ge F^D_i(a^k_i, \ell^k)$ means $a^k_i \ge \hat{\alpha}^D_i(\ell^k)$, and therefore $\alpha^D = \hat{\alpha}^D(\alpha^P) \le \hat{\alpha}^D(\ell^k) \le a^k$. Likewise, $\ell^{k+1}_j \le F^P_j(a^k, \ell^{k+1}_j)$ means $\ell^{k+1}_j \le \hat{\alpha}^P_j(a^k) \le \hat{\alpha}^P_j(\alpha^D) = \alpha^P_j$. Since $\ell^0 = 0 \le \alpha^P$, induction gives $\ell^k \le \alpha^P$ and $\alpha^D \le a^k$ for all k. The upper sequence is symmetric and starts from $\alpha^P \le \bar{F}^P = u^0$. □

Corollary 1 (Local uniqueness)**.** *Suppose that at a parameter value θ_0 the equilibrium α_0 is unique and* $\det(I - DT) \neq 0$ *at α_0. Then there is a neighborhood of θ_0 on which the equilibrium is unique and continuously differentiable in the parameters.*

Proof. The implicit function theorem gives neighborhoods N of θ_0 and V of α^P_0 such that, for $\theta \in N$, the map $T(\cdot; \theta)$ has exactly one fixed point in V, and this fixed point is continuously differentiable in θ. Suppose that, for some sequence $\theta_n \to \theta_0$, $T(\cdot; \theta_n)$ had fixed points outside V. These would lie in the uniformly bounded boxes $[0, \bar{F}^P(\theta_n)]$, and a subsequence would converge to a fixed point of $T(\cdot; \theta_0)$ outside V, contradicting uniqueness at θ_0. □

At the estimate, Corollary 1 justifies treating the continuation values as a differentiable function of the parameters, as the analytic standard errors in Appendix B.4 do.

Implementation. Each iteration solves the doctor-side equations at ℓ^k and at u^k, and then the post-side equations at the resulting doctor-side bounds, by a safeguarded Newton method with relative tolerance 10^{-12}. Each solution x is then moved outward by $10^{-11} \max\{1, |x|\}$. For any component whose inequality in Proposition 3 fails, this margin is multiplied by ten until all

inequalities hold. The iterations stop once successive post-side iterates differ by less than 10^{-10} in every component. The checks use double-precision evaluations of F; they are not carried out in interval arithmetic.

I apply this procedure to seven markets. They are the estimated current market, the current recommendations in the search-retained class, the current fitted exposures in the full-replacement class, and the four best-found long-run allocations of Section 6. Table A.1 reports the results. In all seven markets, after at most 45 iterations, the bracket on every doctor-side and post-side continuation value is narrower than 2.2×10^{-9}. Where the paper reports an equilibrium for the market, that equilibrium lies inside the brackets, and every equilibrium differs from it by at most 1.1×10^{-9} in any component. This is far below the precision of any reported statistic, and I refer to these equilibria as unique. At these equilibria the spectral radius of the Jacobian of T lies between 0.19 and 0.52, so they are also regular and locally stable.

Table A.1. Certified Equilibrium Brackets

		Bracket width		Distance to	
Market	Iterations	Post side	Doctor side	reported	$\rho(DT)$
Current market (both routes behavioral)					
Estimated equilibrium	30 / 35	1.13	2.17	1.09	0.41
Search-retained class (self behavioral, agency fixed)					
Current recommendations	37 / 45	1.98	1.36	1.00	0.52
Long run, current budgets	34 / 41	1.81	1.27	0.92	0.48
Long run, doubled equal budgets	34 / 41	1.68	1.35	0.85	0.49
Full-replacement class (both routes fixed)					
Current fitted exposures	16 / 16	0.80	1.44	–	0.19
Long run, current budgets	34 / 34	1.25	1.48	0.75	0.49
Long run, doubled equal budgets	34 / 34	1.20	1.46	0.75	0.48

Notes: Bracket widths and distances are in units of 10^{-9}. Long run denotes the best-found long-run allocation of Section 6. Iterations are the numbers of steps of the lower and upper sequences in Proposition 3. Bracket widths are the largest componentwise differences between the final upper and lower bounds on post-side and doctor-side continuation values; every equilibrium lies between these bounds. The equilibrium used in the paper lies between these bounds in every market for which one is reported. Distance to reported is the largest absolute difference between either final bound and that equilibrium, in any component, and hence bounds the distance from every equilibrium to the reported one. The current fitted exposures assign each pair its fitted current total exposures $\widehat{M}_{ij}^{\text{self}} + \widehat{M}_{ij}^{\text{agency}}$ through the platform; no equilibrium is reported for this market. $\rho(DT)$ is the spectral radius of the Jacobian of T at the equilibrium.

A.5 Continuous Time and Route Accounting

Independent continuous-time Poisson clocks imply that the probability of a self- and an agency-initiated exposure arriving at exactly the same instant is zero. This justifies adding the two arrival rates. It does not justify pooling their post-exposure outcomes. When acceptance depends on the initiating route, the fixed point must retain

$$\sum_c \lambda_{ij}^c p_{ijc}, \qquad \sum_c \lambda_{ij}^c \psi_{ijc}^D, \qquad \text{and} \qquad \sum_c K_i \lambda_{ij}^c \psi_{ijc}^P$$

as route-weighted sums. The estimator and policy solver use this accounting throughout.

The stationary-renewal assumption enters the structural continuation values and all long-run policy levels. The raw route-specific acceptance patterns do not require it. The short-run objective does not re-solve the stationary system after a policy change, but it still inherits the baseline continuation values from the baseline MPEC estimate. Accordingly, neither the short-run nor the long-run exercise should be read as a literal transition forecast under nonstationary entry, seasonal composition, or permanent depletion of matched types.

B Additional Data and Estimation Material

B.1 Episode-Aware Route and Decision Construction

This subsection reports the post-episode selection rule, treatment of reused post identifiers, raw-media route mapping, bilateral status mapping, and the acceptance exclusions for the cohort defined in Section 2.2. Repeated exposures of a pair within the post's observation period remain distinct events in the Poisson counts. For acceptance, a doctor–post pair whose recorded decision can be attributed to exactly one route in the current post episode contributes at most one Bernoulli trial for each side with an observed decision. I call such pairs *route-attributable*. Pairs with exposures through both routes and pairs whose route cannot be assigned do not enter the route-specific acceptance denominators.

B.2 Descriptive Statistics

Table B.2 summarizes doctor, post, and doctor–post characteristics, while Table B.3 reports the five measures of past doctor–facility activity. Contract history is dated by the recorded contract date; exposure counts, route shares, and recency use the timestamps of the exposures. No outcome resolved after the current post's order date enters the main feature set.

B.3 MPEC Implementation and Convergence Diagnostics

Let $\ell(\theta, \alpha)$ denote the sum of the two route-specific Poisson exposure-count components and the side-specific Bernoulli trials supplied by route-attributable doctor–post pairs, and let $c(\theta, \alpha) =$

Table B.1. Sample, Route, and History Construction

Object	Count	Positive cells	Share (%)
A. Sample flow			
Raw December doctor–post cells	2,768,872		
Timeline-invalid cells excluded	38,488		
Final doctor–post cells	2,730,384		
Active doctors	1,132		
Final posts	2,412		
Timeline-invalid posts excluded	34		
B. Route-specific exposures entering the MPEC likelihood			
Sorting (doctor initiated)	2,503	2,481	0.091
Screening (agent initiated)	1,013	1,009	0.037
C. Prior-relationship support			
All final cells	2,730,384	121,065	4.43
Cells with any panel-recorded exposure	3,518	1,758	49.97
Cells with a recorded bilateral-status decision	3,381	1,716	50.75
Route-attributable acceptance-likelihood cells	3,281	1,685	51.36

Notes: The raw data contain 2,504 doctor-initiated events. The estimation sample excludes one audited out-of-window Web event, recorded on November 18, 2024 after the post's half-open observation window ended on November 15. Consequently, 2,503 doctor-initiated events enter the MPEC likelihood. The same exclusion removes one positive sorting cell and one doctor- and post-side sorting acceptance success/trial. Panel C's first three rows describe the raw final panel; its last row and Panel B describe the post-policy likelihood sample. Repeated in-window exposures remain distinct events. Histories exclude the current post episode and use only information available before its order date.

$\alpha - \Phi(\theta, \alpha)$. For numerical evaluation, the implementation uses the algebraically equivalent update

$$\widetilde{\Phi}_i^D = \frac{\sum_{j,c} \lambda_{ij}^c \{\psi_{ijc}^D + \alpha_i^D p_{ijc}\}}{r + \delta_i^a + h_i^D}, \qquad h_i^D = \sum_{j,c} \lambda_{ij}^c p_{ijc},$$

$$\widetilde{\Phi}_j^P = \frac{\sum_{i,c} K_i \lambda_{ij}^c \{\psi_{ijc}^P + \alpha_j^P p_{ijc}\}}{r + w_j + h_j^P}, \qquad h_j^P = \sum_{i,c} K_i \lambda_{ij}^c p_{ijc}.$$

For either side k, with the corresponding exit rate e_k and match hazard h_k, the residuals obey the pointwise identity

$$(r + e_k + h_k)\{\alpha_k - \widetilde{\Phi}_k(\theta, \alpha)\} = (r + e_k)\{\alpha_k - \Phi_k(\theta, \alpha)\}.$$

The two residual maps therefore have exactly the same zero set. Writing $\widetilde{c}(\theta, \alpha) = \alpha - \widetilde{\Phi}(\theta, \alpha)$, the implemented MPEC problem is

$$\max_{\theta, \alpha} \ \ell(\theta, \alpha) \quad \text{subject to} \quad \widetilde{c}(\theta, \alpha) = 0,$$

which is equivalent to imposing $c(\theta, \alpha) = 0$ in the model. The active parameter space contains 92 structural coordinates, including both route exposure-rate constants, and 3,544 continuation

Table B.2. Market Descriptive Statistics

Variable	N	Mean	SD	P25	Median	P75	Nonzero (%)
A. Doctor characteristics							
Exposures per doctor	1,132	3.11	3.91	—	2.00	—	—
Age	1,132	42.74	12.06	33.00	40.00	50.25	100.00
Experience	1,132	16.49	11.77	7.00	13.00	24.00	100.00
Lagged activity/throughput proxy K_i	1,132	3.70	4.91	1.00	2.00	4.00	100.00
B. Post characteristics							
Exposures per post	2,412	1.46	1.32	—	1.00	—	—
Hours	2,412	10.12	8.68	4.00	8.50	12.50	100.00
Advertised pay	2,412	72.55	43.42	44.00	60.50	91.25	100.00
Advertised wage per hour	2,412	9.13	3.91	5.83	10.00	11.43	100.00
C. Doctor–post characteristics							
Distance	2,730,384	61.08	104.81	22.76	34.40	58.32	100.00
Preferred-location match (%)	2,730,384	25.63	43.66	0.00	0.00	100.00	25.63
Day match (%)	2,730,384	31.49	46.45	0.00	0.00	100.00	31.49
Specialty match (%)	2,730,384	15.84	36.51	0.00	0.00	0.00	15.84

Notes: Entries are raw sample statistics. Indicator variables are shown in percentage points. Age and experience are in years; hours are hours; distance is in kilometers; advertised pay is in thousand yen; and hourly wage is in thousand yen per hour. Dashes mark unavailable quantiles or nonzero shares. The lagged activity measure is a throughput proxy, not observed physical capacity.

Table B.3. Predetermined Doctor–Facility Relationship Variables

Variable	N	Mean	SD	P25	Median	P75	Nonzero (%)
Prior exposures (365 days)	2,730,384	0.11	1.92	0.00	0.00	0.00	2.38
Any prior contract (%)	2,730,384	2.00	13.98	0.00	0.00	0.00	2.00
Days since last exposure	2,730,384	19.58	127.11	0.00	0.00	0.00	4.43
Prior agency share (365 days) (%)	2,730,384	0.49	6.55	0.00	0.00	0.00	0.66
No prior relationship (%)	2,730,384	95.57	20.58	100.00	100.00	100.00	95.57

Notes: History is measured strictly before each post's order date and shown before estimation standardization. Days since last exposure and prior agency share are coded zero when no prior relationship exists; the separate no-prior-relationship indicator records that boundary.

values (1,132 doctor values and 2,412 post values). Fourteen direct coordinates with exact separation or no support are fixed at zero. The calculation does not profile the route constants or impose the fixed-α likelihood score as an additional restriction. No exposure-scale parameter is estimated, so the same continuation values used in acceptance enter the two exposure-rate equations with coefficient minus one.

The parameter search solves the equilibrium conditions at each proposed structural parameter vector and uses derivatives that include the resulting change in continuation values. Proposals are accepted only when they retain equilibrium feasibility and improve the likelihood. The full reduced Hessian is then used to check whether an additional local correction would materially change the parameter vector or likelihood.

The reported estimate has a six-block unprofiled log likelihood of $-28{,}664.909007$, with a maximum absolute equilibrium residual of 2.486900×10^{-13}. At this point the reduced Hessian of the negative log likelihood is positive definite in all 92 active coordinates. The largest normalized Newton correction, computed without a step cap or Hessian regularization, is 2.234477×10^{-5}, and its local quadratic prediction of the increase in total log likelihood is 1.553244×10^{-10}.[8] Together with two successive small updates accepted without truncation, these values satisfy the practical numerical criteria: an equilibrium residual at most 10^{-8}, a remaining normalized correction below 0.001, and a predicted likelihood gain below 0.01. The final curvature also passes directional finite-difference checks. Table B.10 reports the numerical diagnostics. This is a local numerical convergence assessment; it does not establish a global optimum of the likelihood. Uniqueness of the equilibrium at the estimate is established separately in Appendix A.4.

B.4 Analytic Standard Errors

The standard errors allow the continuation values to respond to changes in the structural parameters. Let $f = -\ell$ be the raw total negative log likelihood. At the estimate the equilibrium is unique and regular (Appendix A.4), so by Corollary 1 the continuation values are a differentiable function of the parameters near the estimate. Define

$$D = \frac{\partial \alpha}{\partial \theta'} = -\widetilde{c}_\alpha^{-1}\widetilde{c}_\theta, \qquad \nu = -\widetilde{c}_\alpha^{-T} f_\alpha, \qquad R = \begin{bmatrix} I \\ D \end{bmatrix}.$$

All quantities are evaluated at the reported estimate. The observed curvature of the likelihood restricted to the equilibrium branch is

$$H = R'\nabla^2_{(\theta,\alpha)}\{f(\theta,\alpha) + \nu'\widetilde{c}(\theta,\alpha)\}R,$$

[8] For the numerical step diagnostic, write $\theta_k = s_k u_k$, where $s_k = 1/\max\{1, \sqrt{\text{mean}(z_k^2)}\}$ and the mean is taken over doctor–post pairs for the corresponding design column. The two initiator-specific acceptance intercept differences have $s_k = 1$. Reported coefficients and their covariance retain the original parameter coordinates.

where ν is held fixed when taking the Hessian. Including the curvature of the constraints is necessary; holding continuation values fixed would give a different information matrix.

For doctor–post pair (i, j), let ℓ_{ij} collect its two exposure-count contributions and all observed acceptance-decision contributions. Its score on the equilibrium branch is

$$s_{ij} = \nabla_\theta \ell_{ij} + D' \nabla_\alpha \ell_{ij}.$$

Following Cameron, Gelbach and Miller (2011), I cluster these scores by doctor and by post, allowing dependence among pairs sharing either index. Writing $S_i^D = \sum_j s_{ij}$ and $S_j^P = \sum_i s_{ij}$, the two-way cluster covariance uses

$$B = \sum_i S_i^D (S_i^D)' + \sum_j S_j^P (S_j^P)' - \sum_{i,j} s_{ij} s_{ij}', \qquad \widehat{V} = H^{-1} B H^{-1}.$$

The subtraction removes double counting of the doctor–post intersections. The main calculation uses no finite-cluster correction (CR0). Its covariance is positive semidefinite and all 92 marginal variances are positive, so no covariance adjustment is used. Model-based observed-information standard errors and a finite-cluster correction (CR1) are retained as supplementary diagnostics.[9] Inference conditions on the calibrated hazards and discount rate, the activity proxies, and observed covariates and market composition. It does not incorporate uncertainty in those calibrations or allow arbitrary dependence across pairs sharing neither a doctor nor a post.

Probability changes and approximate standard errors. Table 2 translates selected coefficients into average probability changes with continuation values and other covariates held fixed. For acceptance, let $F(v) = \Lambda(v)$ and let $\widehat{\eta}_{ij}$ be the fitted acceptance index for the relevant route and side. For exposure, let $F(v) = 1 - \exp\{-\exp(v)\}$ and $\widehat{\eta}_{ij} = \log m_{ij}^c$, including the observed $K_i T_j$ offset. For a standardized continuous covariate k, set $v_{ij,k} = \widehat{\eta}_{ij}$; for an indicator, set $v_{ij,k} = \widehat{\eta}_{ij} - \widehat{\beta}_k z_{ij,k}$ to obtain the index when that indicator is zero. With averaging weights w_{ij} summing to one, define

$$g_k(b) = 100 \sum_{i,j} w_{ij} \{F(v_{ij,k} + b) - F(v_{ij,k})\}.$$

The reported probability change is $g_k(\widehat{\beta}_k)$. Exposure comparisons weight all retained doctor–post pairs equally; acceptance comparisons weight the observed decisions in the relevant route and side. The two relationship-history comparisons restrict these samples to pairs with a positive exposure count in the preceding 365 days, keeping the original estimation-sample standardization.

[9] The model-based covariance is H^{-1}. The CR1 sensitivity multiplies each of the three cluster sums by $[G/(G-1)][(N-1)/(N-92)]$, using its own number of groups G. Here N = 2,730,384 doctor–post pairs, with 1,132 doctor groups, 2,412 post groups, and N intersection groups. The scores are not demeaned.

For a simple uncertainty summary, I apply the delta method to each coefficient separately, treating the starting indices $v_{ij,k}$ and the weights as fixed:

$$\widehat{\text{se}}\{g_k(\widehat{\beta}_k)\} \simeq 100 \left| \sum_{i,j} w_{ij} F'(v_{ij,k} + \widehat{\beta}_k) \right| \widehat{\text{se}}(\widehat{\beta}_k).$$

Here $F'(v) = \Lambda(v)\{1 - \Lambda(v)\}$ for acceptance and $F'(v) = \exp(v)\exp\{-\exp(v)\}$ for exposure. The coefficient standard errors on the right-hand side are the doctor–post clustered standard errors derived above. This approximation propagates uncertainty in the coefficient being translated, but omits uncertainty in the starting predictions and covariance with other estimated parameters.

B.5 Covariate Coefficient Estimates

Table B.4 reports the selected index coefficients underlying Table 2.

Table B.4. Selected Parameter Estimates

	Sorting	Screening	Doctor acc.	Post acc.
Log distance	-0.4190	-0.1945	0.1002	0.0063
	(0.0442)	(0.0761)	(0.1275)	(0.0655)
Hours	0.0311	0.4596	-0.3822	0.0896
	(0.0674)	(0.3018)	(0.1918)	(0.3526)
Log posted salary	-0.1151	-2.1926	-0.0877	-2.0443
	(0.0525)	(0.5228)	(0.1780)	(0.4999)
Preference match	0.2417	0.1929	0.1489	-0.0437
	(0.0851)	(0.1519)	(0.2653)	(0.1418)
Day match	0.9223	0.3076	0.8349	0.2153
	(0.1291)	(0.1351)	(0.2472)	(0.1305)
Specialty match	0.6213	0.9296	-0.3856	0.3381
	(0.0731)	(0.1823)	(0.2882)	(0.1841)
Prior exposures (past 365 days)	0.0448	0.0485	0.0552	-0.0170
	(0.0100)	(0.0076)	(0.0226)	(0.0122)
Prior contract	0.8902	2.9306	-1.0797	1.3925
	(0.1075)	(0.2756)	(0.6210)	(0.2941)

Notes: Entries are raw index coefficients. Continuous covariates are standardized; indicator coefficients correspond to a change from zero to one. Analytic standard errors in parentheses are clustered by doctor and post (two-way CR0). Scores include the response of continuation values through the equilibrium conditions. Inference conditions on the maintained equilibrium branch, calibrated hazards and discount rate, the observed market composition, and activity proxies. Calibration uncertainty is not included.

The full tables report the 90 active coefficients in the four exposure and acceptance indices, including their intercepts, and a separate panel reports the two initiator-specific acceptance intercept differences. Standard errors appear in parentheses beneath the estimates. Continuous covariates are standardized, while indicators retain their zero-to-one scale. Fourteen unsupported coordinates are held at zero and shown as dashes, rather than as estimated zeros. The exposure loadings on continuation values are fixed normalizations rather than estimated coefficients.

Table B.5. Parameter Estimates: Doctor acceptance

Covariate	Estimate (SE)
Intercept	17.4105 (1.9456)
Log distance	0.1002 (0.1275)
Doctor age	4.0249 (4.0325)
Experience	-2.3576 (3.7282)
Hours	-0.3822 (0.1918)
Log posted salary	-0.0877 (0.1780)
Preference match	0.1489 (0.2653)
Day match	0.8349 (0.2472)
Specialty match	-0.3856 (0.2882)
Outpatient match	0.1075 (0.2991)
Emergency match	–
Home-visit match	0.2504 (0.7098)
Ward match	-0.8459 (0.3551)
Dialysis match	-0.3295 (0.6658)
Health-check match	0.3603 (0.5175)
Endoscopy match	0.2761 (0.6401)
Surgery match	–
House-call match	-0.5780 (0.6325)
Work-with-peer match	–
Flexible-work match	-0.3642 (0.8010)
Occupational-health match	–
Prior exposures (past 365 days)	0.0552 (0.0226)
Prior contract	-1.0797 (0.6210)
Days since last exposure	-0.1643 (0.0606)
Prior agency share (past 365 days)	0.0021 (0.0288)
No prior relationship	-0.9899 (0.6278)

Notes: Entries are raw index coefficients; continuous covariates are standardized and indicators are not. Dashes denote fixed unsupported coordinates, not estimated zeros. Exposure loadings on continuation values are normalized to one and are not estimated rows. Analytic standard errors in parentheses are clustered by doctor and post (two-way CR0). Scores include the response of continuation values through the equilibrium conditions. Inference conditions on the maintained equilibrium branch, calibrated hazards and discount rate, the observed market composition, and activity proxies. Calibration uncertainty is not included.

Table B.6. Parameter Estimates: Post acceptance

Covariate	Estimate (SE)
Intercept	6.8671 (0.6760)
Log distance	0.0063 (0.0655)
Doctor age	-0.1571 (0.1912)
Experience	-0.1059 (0.1870)
Hours	0.0896 (0.3526)
Log posted salary	-2.0443 (0.4999)
Preference match	-0.0437 (0.1418)
Day match	0.2153 (0.1305)
Specialty match	0.3381 (0.1841)
Outpatient match	0.0064 (0.1832)
Emergency match	–
Home-visit match	-1.0348 (0.4614)
Ward match	-0.0805 (0.2175)
Dialysis match	0.2633 (0.6531)
Health-check match	-0.0768 (0.3434)
Endoscopy match	0.7435 (0.4932)
Surgery match	–
House-call match	-1.1422 (0.4684)
Work-with-peer match	0.0096 (0.5326)
Flexible-work match	-2.7982 (0.4949)
Occupational-health match	–
Prior exposures (past 365 days)	-0.0170 (0.0122)
Prior contract	1.3925 (0.2941)
Days since last exposure	-0.0879 (0.0460)
Prior agency share (past 365 days)	-0.0417 (0.0242)
No prior relationship	-0.2559 (0.2999)

Notes: Entries are raw index coefficients; continuous covariates are standardized and indicators are not. Dashes denote fixed unsupported coordinates, not estimated zeros. Exposure loadings on continuation values are normalized to one and are not estimated rows. Analytic standard errors in parentheses are clustered by doctor and post (two-way CR0). Scores include the response of continuation values through the equilibrium conditions. Inference conditions on the maintained equilibrium branch, calibrated hazards and discount rate, the observed market composition, and activity proxies. Calibration uncertainty is not included.

Table B.7. Parameter Estimates: Sorting exposure

Covariate	Estimate (SE)
Intercept	10.8022
	(1.7046)
Log distance	-0.4190
	(0.0442)
Doctor age	2.5331
	(3.8711)
Experience	-0.9331
	(3.5753)
Hours	0.0311
	(0.0674)
Log posted salary	-0.1151
	(0.0525)
Preference match	0.2417
	(0.0851)
Day match	0.9223
	(0.1291)
Specialty match	0.6213
	(0.0731)
Outpatient match	0.3790
	(0.0940)
Emergency match	–
Home-visit match	1.1057
	(0.2269)
Ward match	0.3420
	(0.1190)
Dialysis match	2.3813
	(0.2438)
Health-check match	0.5623
	(0.1108)
Endoscopy match	1.5732
	(0.3521)
Surgery match	–
House-call match	0.2291
	(0.4071)
Work-with-peer match	1.5811
	(0.5591)
Flexible-work match	1.8620
	(0.2457)
Occupational-health match	–
Prior exposures (past 365 days)	0.0448
	(0.0100)
Prior contract	0.8902
	(0.1075)
Days since last exposure	-0.2692
	(0.0293)
Prior agency share (past 365 days)	-0.0475
	(0.0119)
No prior relationship	-2.5000
	(0.1253)

Notes: Entries are raw index coefficients; continuous covariates are standardized and indicators are not. Dashes denote fixed unsupported coordinates, not estimated zeros. Exposure loadings on continuation values are normalized to one and are not estimated rows. Analytic standard errors in parentheses are clustered by doctor and post (two-way CR0). Scores include the response of continuation values through the equilibrium conditions. Inference conditions on the maintained equilibrium branch, calibrated hazards and discount rate, the observed market composition, and activity proxies. Calibration uncertainty is not included.

Table B.8. Parameter Estimates: Screening exposure

Covariate	Estimate (SE)
Intercept	-2.2979 (0.7053)
Log distance	-0.1945 (0.0761)
Doctor age	-0.5681 (0.2376)
Experience	0.4081 (0.2374)
Hours	0.4596 (0.3018)
Log posted salary	-2.1926 (0.5228)
Preference match	0.1929 (0.1519)
Day match	0.3076 (0.1351)
Specialty match	0.9296 (0.1823)
Outpatient match	-0.0060 (0.1919)
Emergency match	–
Home-visit match	-0.3495 (0.5036)
Ward match	-0.0708 (0.1668)
Dialysis match	2.6438 (0.5978)
Health-check match	0.3370 (0.3002)
Endoscopy match	1.3536 (0.4886)
Surgery match	–
House-call match	0.4815 (0.4785)
Work-with-peer match	–
Flexible-work match	1.7356 (0.3958)
Occupational-health match	–
Prior exposures (past 365 days)	0.0485 (0.0076)
Prior contract	2.9306 (0.2756)
Days since last exposure	-0.2565 (0.0755)
Prior agency share (past 365 days)	0.1412 (0.0124)
No prior relationship	-1.3779 (0.3390)

Notes: Entries are raw index coefficients; continuous covariates are standardized and indicators are not. Dashes denote fixed unsupported coordinates, not estimated zeros. Exposure loadings on continuation values are normalized to one and are not estimated rows. Analytic standard errors in parentheses are clustered by doctor and post (two-way CR0). Scores include the response of continuation values through the equilibrium conditions. Inference conditions on the maintained equilibrium branch, calibrated hazards and discount rate, the observed market composition, and activity proxies. Calibration uncertainty is not included.

Table B.9. Initiator-Specific Acceptance Intercept Differences

Route-specific parameter	Estimate (SE)
Doctor-initiated acceptance intercept difference	5.8706 (0.3758)
Agent-initiated acceptance intercept difference	3.8459 (0.2206)

Notes: These two parameters are active components of the 92-coordinate structural vector. Analytic standard errors in parentheses are clustered by doctor and post (two-way CR0). Scores include the response of continuation values through the equilibrium conditions. Inference conditions on the maintained equilibrium branch, calibrated hazards and discount rate, the observed market composition, and activity proxies. Calibration uncertainty is not included.

B.6 Numerical Diagnostics

Table B.10. Estimation and Analytic Inference Diagnostics

Diagnostic	Value
Active / fixed coordinates	92 / 14
Total log likelihood	-28,664.909007
Equilibrium residual (maximum absolute value)	2.487e-13
Remaining normalized Newton correction (maximum)	2.234e-05
Local quadratic predicted log-likelihood gain	1.553e-10
Practical local numerical criteria	Satisfied
Doctor / post clusters	1,132 / 2,412
Doctor–post intersections	2,730,384
Primary analytic covariance	Two-way CR0
Primary covariance positive semidefinite	Yes

Notes: The remaining correction uses the full equilibrium-reduced Hessian without regularization. The likelihood gain is a local quadratic prediction, not a bound on global improvement. Numerical convergence does not establish a global optimum or equilibrium uniqueness. Analytic standard errors in parentheses are clustered by doctor and post (two-way CR0). Scores include the response of continuation values through the equilibrium conditions. Inference conditions on the maintained equilibrium branch, calibrated hazards and discount rate, the observed market composition, and activity proxies. Calibration uncertainty is not included.

C Additional Policy Results

Figure C.1 displays the access–value comparison with doctor search retained. Every long-run candidate in this comparison recomputes doctor search from the solved continuation values.

C.1 Who Gains from the Policy Changes?

Table C.1 separates the value changes on the two sides of the market for the current-budget comparisons and reports how broadly each side's gains are shared. Panel A of Table 5 reports the corresponding results for recommendation expansion, equalization across doctors, and pairwise redesign. In both tables, doctor-side value is $\sum_i K_i \alpha_i^D$, so the reported winning share weights doctors by their numbers of slots. Post-side value is $\sum_j \alpha_j^P$, and each post receives equal weight. A unit is classified as gaining or losing when its continuation value changes by more than 10^{-4} in the corresponding direction; smaller changes are classified as unchanged. These shares describe the distribution of the model's predicted value changes and are not sampling probabilities.

For the group comparisons in the text and below, I rank posts by their continuation values under the current hybrid and divide them into five approximately equal-sized groups using their cumulative stock weights. Group membership remains fixed across every policy comparison. Each reported group percentage is the change in that group's total value relative to its value under the preceding policy in the comparison. Thus, the expanded-budget redesign comparison uses the doubled, equal-budget profile as its denominator while retaining the groups defined under the current hybrid. This distinguishes initially low-value posts from posts selected after observing the policy gain.

Table C.1. Who Gains from Current-Budget Recommendation Policies?

	Change in continuation value			Share with higher value (%)	
Policy comparison	Doctors	Posts	Total	Doctor slots	Posts
Current to search-retained redesign	−155.04	+1,218.64	+1,063.60	31.36	45.85
Current to full replacement	−8,823.99	+8,003.08	−820.91	1.08	100.00

Notes: Each row compares the final allocation with the current hybrid. Doctor-side value weights each doctor by the number of doctor slots; post-side value sums over posts. Share with higher value is the percentage of doctor slots or posts whose continuation value exceeds its current-hybrid value by more than 10^{-4}. Doctors are weighted by their numbers of slots; posts receive equal weight. These are accounting comparisons conditional on the estimated model. They do not imply that every unit on a gaining side benefits.

Uneven gains across the three steps. Panel A of Table 5 shows that the three steps of Section 6.4 distribute their gains differently between doctors and posts. Relative to the current

hybrid, the final allocation lowers doctor-side value by 1.03 percent while raising post-side value by 19.68 percent. Continuation values increase for 17.47 percent of doctor slots and 70.44 percent of posts. Expansion and equalization raise post-side value while reducing doctor-side value. Pairwise redesign then recovers part of those doctor-side losses: it raises doctor-side value by 1.60 percent and post-side value by 2.50 percent. Doctors receive 69.52 percent of this last increment. In this final step, continuation values increase for 97.18 percent of doctor slots and 57.88 percent of posts. Among posts, the lowest baseline-value quintile gains 11.17 percent, while the highest quintile loses 4.94 percent. These comparisons show why the aggregate increments should be assessed together with who receives them.

C.2 Continuation Value and Accepted Matches

The number of matches alone does not determine the value of a recommendation policy. An accepted match generates a gain above continued waiting for each side, and the continuation-value equations aggregate these gains after accounting for discounting and exogenous exit. The following accounting separates changes in the number of matches from changes in who matches and the gains obtained in those matches. All quantities are evaluated at each policy's equilibrium, rather than at the fixed continuation values used in the short-run objective.

Let $m_{ijc} = M^c_{ij} P^D_{ijc} P^P_{jic}$ be the expected number of accepted matches for doctor–post pair (i, j) through route c. Independence of the two payoff shocks implies that each side's expected gain above continued waiting, conditional on a match, is

$$g^D_{ijc} = \frac{q^D_{ijc}}{P^D_{ijc}} = \mathbb{E}[U^c_{ij} - \alpha^D_i \mid U^c_{ij} \geq \alpha^D_i],$$
$$g^P_{ijc} = \frac{q^P_{ijc}}{P^P_{jic}} = \mathbb{E}[V^c_{ji} - \alpha^P_j \mid V^c_{ji} \geq \alpha^P_j].$$

Multiplying the doctor equation in (5) by K_i and summing both sides of the market gives

$$W = \sum_{i,j,c} m_{ijc} \left(\frac{g^D_{ijc}}{r + \delta^a_i} + \frac{g^P_{ijc}}{r + w_j} \right).$$

Thus, a match contributes more to this accounting when its conditional gain above continued waiting is larger. The gains are measured in the model's utility units; they are neither wages nor gross match payoffs. The calibration sets $r = -\log(0.99)$, $\delta^a_i = 0.05$ for every doctor, and $w_j = 0.10$ for every post. These rates are fixed across policies and constant within each side, so changes in match composition do not operate through differences in exit rates within a side. The different rates across sides do matter: an equal surplus flow receives a larger continuation-value weight on the doctor side.

To distinguish more matches from different matches, let $Q = \sum_{i,j,c} m_{ijc}$, $s_{ijc} = m_{ijc}/Q$, and $v_{ijc} = g^D_{ijc}/(r + \delta^a_i) + g^P_{ijc}/(r + w_j)$. Then $W = Q\mu$, where $\mu = \sum_{i,j,c} s_{ijc} v_{ijc}$ is the average

Table C.2. Match Counts, Match Composition, and Continuation Value

			Contributions to value change		
Comparison	Match change	Value change	Match number	Composition	Conditional gains
Current to search-retained redesign	+100.77	+1,063.60	+7,987.97	−5,123.50	−1,800.87
Short-run to long-run rule	+81.70	+1,295.54	+6,413.12	−5,122.38	+4.80
Current to full replacement	+692.02	−820.91	+46,014.76	−60,438.67	+13,603.00
Redesign at doubled equal budgets	+167.08	+1,717.40	+11,628.29	−3,783.03	−6,127.85

Notes: Values are in model utility units. The last three columns use the symmetric accounting in equation (12): match number, the distribution of accepted matches across doctor–post pairs and routes, and conditional gains above continued waiting at each pair and route. Their sum reproduces the change in the equilibrium flow representation of value. Before rounding, the largest difference from the saved continuation-value change is below 0.003, due to the equilibrium tolerance used for the fixed comparison policies. Components are accounting terms, not effects of independently changing one behavioral channel.

contribution per accepted match. For any two policies, write Δ for the after-minus-before difference and an overbar for the mean of the two endpoint values. Applying the symmetric two-factor identity first to $Q\mu$ and then to each $s_{ijc}v_{ijc}$ gives

$$\Delta W = \underbrace{\Delta Q\,\overline{\mu}}_{\text{match number}} + \underbrace{\overline{Q}\sum_{i,j,c}\Delta s_{ijc}\,\overline{v}_{ijc}}_{\text{match composition}} + \underbrace{\overline{Q}\sum_{i,j,c}\overline{s}_{ijc}\,\Delta v_{ijc}}_{\text{conditional gains}}. \tag{12}$$

The second term changes which doctor–post pairs and routes generate the matches, evaluating their contributions at the average of the two endpoint values. The third changes the conditional gains at a fixed average match composition. This is an exact accounting identity at equilibrium, not a sequence of independently implementable policy interventions. Conditional gains themselves subtract policy-specific continuation values, so the last term is not an estimate of a change in gross match quality.

Table C.2 reports these terms. Components are rounded independently, so the displayed numbers need not add exactly; the reported numerical residuals refer to the unrounded calculations.

The search-retained long-run rules increase both matches and value relative to their respective current, short-run, or equal-budget comparison policies. In each comparison, the positive match-number contribution exceeds the combined change from match composition and conditional gains. Full replacement, by contrast, generates more matches but lower total value than the current hybrid: its negative match-composition contribution more than offsets the positive match-number and conditional-gain contributions. Thus, the number of matches alone does not determine the value ranking; which doctor–post pairs match and through which routes also matters.

To understand the 1.80 percent value gain from the final pairwise redesign of Section 6.4,

I examine how exposures and accepted matches change through doctor search and recommendations. Panel B of Table 5 compares the doubled, equal-budget profile with the redesigned allocation. Doctor-initiated exposures fall by 25.40 percent, while matches through that route rise by just 0.09 percent. The share of these exposures that both sides accept rises from 16.26 to 21.81 percent. At the same time, the fixed number of recommendations generates 29.94 percent more matches, and their conversion rate rises from 24.42 to 31.73 percent. Thus, the model predicts more successful recommendations while fewer doctor applications sustain roughly the same number of doctor-initiated matches.

The exposure and acceptance margins explain how the expanded-budget redesign can reduce doctor search while preserving doctor-initiated matches. For each pair and route, the match count is the product of exposures, doctor acceptance, and post acceptance. I allocate its change to these three factors by replacing them in every possible order and averaging the marginal contributions over the six orders. Within doctor search, changed exposure counts contribute -211.86 matches, changes in post acceptance contribute 215.58, and changes in doctor acceptance contribute -3.09. The near cancellation is consistent with the almost unchanged doctor-initiated match count reported above. These terms compare equilibrium endpoints; holding only one factor fixed need not yield another equilibrium and does not identify the causal effect of separately changing that factor.

C.3 Numerical Policy Search

The long-run searches stop when no feasible change in the recommendation matrix offers a sufficiently large first-order improvement in total user value. For policy class k and its fixed budget vector b, let $g^k(A) = \nabla_A W^k(A; K)$ include the response of the equilibrium to A. Define

$$G^k(A) = \max_{B \in \mathcal{B}^k(b)} \sum_{i,j} g^k_{ij}(A)(B_{ij} - A_{ij}).$$

The maximization ranges over the complete feasible policy set, including pairs that currently receive no recommendations. I stop when $G^k(A)/\max\{1, |W^k(A; K)|\} \leq 10^{-6}$ and require a maximum equilibrium residual no greater than 10^{-10}. All four reported long-run allocations satisfy these criteria, as shown in Table C.3. The gradient is obtained from the equilibrium equations; its adjoint solve must have relative residual no greater than 10^{-9}.

Independent checks reproduce each allocation's value, feasibility, and first-order gap, and compare the gradient with finite differences along feasible directions. Initializing the equilibrium solver at either the reported continuation values or those of a different policy reproduces the same values. These checks support approximate first-order stationarity. They do not establish a global policy optimum or a bound on the remaining global value gain. Appendix A.4 certifies that the equilibrium at each of the four allocations is unique. The fixed comparison policies and

Table C.3. Numerical Checks for the Long-Run Policies

Policy class	Doctor budgets	Total value	Relative gap	Residual	Audit
Search retained	Current	95,478.79	9.07e-07	1.36e-14	Pass
Search retained	Doubled, equal	97,166.10	9.86e-07	2.67e-14	Pass
Full replacement	Current	93,594.29	8.90e-07	2.92e-15	Pass
Full replacement	Doubled, equal	94,188.29	9.97e-07	4.23e-14	Pass

Notes: Each policy satisfies a relative Frank–Wolfe gap of at most 10^{-6} on its fixed feasible set. The gap is the maximum directional derivative toward another feasible policy, divided by total value. Equilibrium residuals are at most 10^{-10} and relative adjoint residuals at most 10^{-9}. Independent audits check equilibrium replay, a separately assembled gradient, and feasible-direction finite differences. These checks certify approximate first-order stationarity, not global optimality or equilibrium uniqueness. Fixed comparison policies and market-expansion grid equilibria use an equilibrium tolerance of 10^{-5}.

the market-expansion grid are solved to an equilibrium tolerance of 10^{-5}; the tighter residual criterion applies to the four optimized long-run allocations.

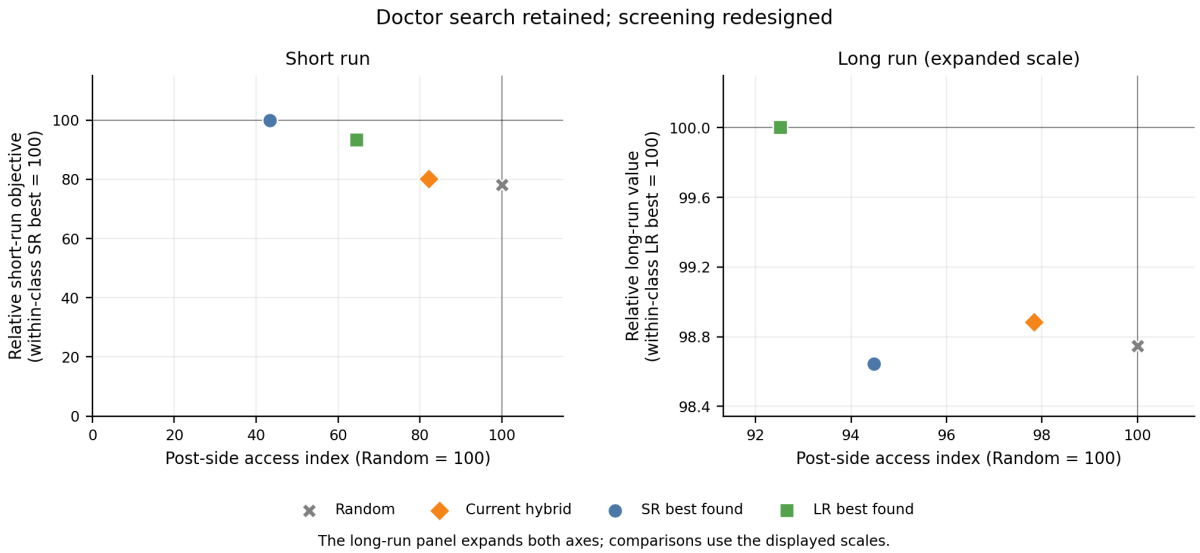

Figure C.1. Access and value when agent screening is redesigned and doctor search responds in equilibrium.