Joint Venture Formation in Procurement Auctions*

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Abstract

We investigate the impact of joint ventures on the efficiency of procurement auctions. To explore the optimal utilization of joint ventures, we must capture their two opposing competitive effects and identify the joint venture formation process. For this purpose, we develop and estimate a two-stage structural model. In the first stage, we determine the distribution of the number of joint ventures and single bidders. In the second stage, bidders compete in an asymmetric scoring auction, where the asymmetry arises from the type difference between joint ventures and single bidders. Our estimation results reveal the existence of cost synergies associated with joint ventures but also identify two significant obstacles to their formation: adjustment costs and search frictions. Through simulations, we demonstrate that while moderate government support for joint venture formation can enhance procurement efficiency, excessive support may reduce it by diminishing competition. These findings highlight the need for balanced policy interventions to optimize the benefits of joint ventures in procurement auctions.

JEL Classification Codes: L24; D22; D44; H57.

Keywords: Joint ventures; Cost synergies; Matching; Procurement auctions.

1 Introduction

In many auctions, the auctioneer set some condition under which the potential entrants are allowed to form a joint venture (JV hereafter) and to submit a joint bid. This system,

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which at first glance appears to reduce the number of bidders and potentially prohibit the competitive environment, is actually adopted to address various challenges encountered in many real-world auctions. The most well-known example is common value auctions, such as Outer Continental Shelf (OCS) auctions (Hendricks and Porter, 1992). Classical literature highlights that allowing joint ventures facilitates information pooling, enabling more efficient auctions. In Official Development Assistance (ODA) procurement auctions (Iimi, 2004), it has been pointed out that joint ventures can increase the budget scale, allowing more entrants in auctions where there are strict budget constraints. More recently, in patent auctions (Asker, Baccara and Lee, 2021) and procurement auctions (Bouckaert and Van Moer, 2021), it has been noted that the effect of efficient reallocation—where more efficient entities are able to execute projects in response to idiosyncratic cost shocks—improves project efficiency.

In many regions, the formation of JVs is conditionally permitted in public procurement auctions[1][2]. However, there are two significant hurdles in utilizing the JV system to achieve the efficient procurement. The first issue arises from the fact that JVs can have anti-competitive aspects. This effect occurs through two channels: the direct reduction in the number of bidders due to the formation of a JV, and the indirect path, where the possibility of a JV weakens the entry incentive of the potential entrants because JVs are likely to be cost-effective in some form. The appropriate scale of JV utilization is determined by the balance between its pro-competitive and anti-competitive effects: simply encouraging its use is not sufficient, and the sophisticated policy management is required.

The second hurdle is that allowing JVs makes it impossible for the procurer to fully control the entry behavior of potential entrants. Before a public procurement, potential entrants engage in free interaction to form JVs. This process is informal, without any platform, matching system, or group formation mechanism in place. As a result, it becomes extremely difficult for the procurer to predict what entry patterns will emerge under different auction settings. This implies that, even if an ideal JV participation rate is determined in some way, it remains unclear what measures can be taken to realize that desired entry pattern.

[1]For instance, only 5% of bids are submitted by joint ventures in Europe (Gugler, Weichselbaumer and Zulehner, 2021), whereas 24% of bids in Japanese procurement auctions come from joint ventures when considering only the largest auctions.

[2]According to Bouckaert and Van Moer (2021), in antitrust cases in Nordic countries, two criteria are used to address these two effects: the no-solo-bidding test and efficiency pass-through.

In this paper, we develop and estimate a two-stage structural model to analyze procurement auctions with joint venture formation[3]. In the first stage, we model the entry decisions and joint venture formation process based on the interactions between potential entrants. In the second stage, we consider an auction to determine the service provider for the project. The model allows us to identify the joint venture formation process, enabling us to simulate counterfactual auction settings and explore optimal interventions to maximize procurement efficiency. Whereas our estimates reveal that a efficiency improvement by forming a JV, our simulation predicts that the excessive encouragement of JVs might reduce the efficiency of the procurement. This finding is due to the less bidders caused by the more likelihood of the successful JV formation.

Our empirical context involves Japanese procurement auctions held by five Regional Development Bureaus under the Ministry of Land, Infrastructure, Transport, and Tourism (MLIT), which are responsible for developing and maintaining public infrastructure directly managed by the MLIT. For large-scale projects, these offices allow potential entrants to form a type of joint venture called a "project-specific JV" (Tokutei JV in Japanese). The auction winners are determined through a scoring auction, where project proposals are scored according to a pre-determined rule, and bidders compete based on the ratio of their score to their submitted bid. We have compiled a new dataset of all auctions held by these offices from 2006 to 2022, focusing on two types of construction projects where JVs are common: Civil Engineering and Bridge Building. The dataset includes 793 auctions, for which we can observe the scores and bids submitted by all bidders, the auction winners, and whether each bidder is a JV. Additionally, for each auction, we have access to the engineer's estimate of the expected project cost, which is used to determine the size of the project.

We overview the model in reverse order. In the second stage, which we refer to as the bidding stage, we model an asymmetric scoring auction. In this auction, the pair of project cost and the score of the submitted plan constitutes the bidder's type. The asymmetry arises from potential differences in the type distributions between the two entry modes: JV and individual bidders. We expect that JVs are more likely to draw stronger types than

[3]Importantly, an integrated model is required to capture the indirect anti-competitive effect—specifically, how the likelihood of competing against JVs in the auction stage affects the incentive to enter.

individual bidders, which we interpret as resulting from *cost synergies*. Although this is a two-dimensional private information game, the structure of the problem allows us to solve the optimal bidding strategy using a one-dimensional object: the ratio of the score to the cost.

In the first stage, which we refer to as the entry and joint venture formation stage, we start by modeling the individual decision problem over *entry intentions*. Each potential entrant chooses one of the following intentions: (1) forming a JV, (2) entering as a single bidder, or (3) not entering. These intentions may not directly determine the entry mode: even if a potential entrant chooses to form a JV, they might not succeed in partnering with another potential entrant. We then directly parametrize the mapping from the distribution of the set of intentions of potential entrants to the distribution of the entry patterns. Instead of building a fully structural model to describe group formation, our semi-parametric approach avoids the risk of mis-specification inherent in detailed structural modeling, while still allowing us to simulate market outcomes.

We argue that the mapping from the distribution of the set of intentions to the distribution of the entry patterns is recovered from our data. To identify the mapping, the marginal distribution of the number of JVs is enough. Imagine a mapping, denoted by A, yields the observed marginal distribution of the number of JVs. While the detail is left to Section 4.2.1, under this mapping, there is unique corresponding choice probability of the intention of forming a JV. When we pick any different mapping, which is denoted by B, it is natural that this new mapping yields the different distribution of the number of JVs under the same choice probability. Can we renew the choice probability to generate the same distribution? No, because, under the same choice probability, for at least one combination of JV counts (M_1, M_2), higher probability is assigned to M_1 in A, and to M_2 in B. Hence, no new choice probability can simultaneously align both at the same time.

In the first stage, we find that adjustment costs and search friction hinder JV formation. In particular, the estimated search friction suggests that at least four firms must choose the intention of trying to form a joint venture to observe at least one joint venture in a given auction. In other words, there is the possibility of failing to find a partner when forming a JV. Moreover, our estimates suggest that, compared to entering as a single bidder, managing a joint venture additionally requires approximately 6 million Japanese yen, which constitutes 23.72% of the expected payoff in small-scale procurement auctions. In the second stage, our recovered distributions of cost types show a clear difference between single bidders and JVs. Particularly, JVs are likely to draw a more competitive type than single bidders, implying the existence of cost synergy.

In a counterfactual scenario where we simulate the promotion of joint venture formation by reducing the adjustment cost, we find that a mild level of promotion leads to lower procurement costs, whereas excessive promotion results in higher procurement costs. This non-linearity arises from the transition of the relative strengths of the pro- and anti-competitive effects. Excessive promotion leads to a growing number of joint ventures which reduces the number of bidders and affects the entry incentives of others. This anti-competitive effect eventually overwhelms the pro-competitive effect generated by cost synergy.

1.1 Related Literature

The first strand of the literature to which this study contributes is the empirical literature on joint venture formation and endogenous merger formation. Forming a joint venture and mergers share some similarities in that parties commit to making collective decisions. The competitive effect of joint ventures has been studied intensively in the merger literature including Shapiro and Willig (1990), Estache and Iimi (2009), and Miller and Weinberg (2017). The incentives for mergers and for joint venture formation are complex due to externalities in subsequent competition, rendering it challenging to build a tractable and internally-consistent model. The literature has developed in two strands: (i) building a long-term horizon dynamic model, as seen in Gowrisankaran (1999) and Igami and Uetake (2020), and (ii) utilizing the matching model developed by Fox (2018), as shown in Akkus, Cookson and Hortaçsu (2016) and Uetake and Watanabe (2020).

This study proposes a new tractable method to estimate an endogenous joint venture/merger formation model by extending the entry model developed by Seim (2006) and estimating the joint venture formation pattern directly from the data. Our generalization allows the situation where the individual choice might not be equal to the observed individual market outcome: in our case, the single bidder in a procurement auction might be the potential entrant who initially tries to form a joint venture. We provide a formal estimation procedure for this model using an MPEC (Su and Judd, 2012). Another perspective on our methodology is illustrated by comparing with the lottery choice model discussed in Agarwal and Somaini (2018, 2020). They also consider situations where individual choices do not directly determine individual outcomes, such as in the school choice problem where the assignment is determined by a pre-defined mechanism like the deferred acceptance (DA) algorithm. Our work diverges from theirs in that we do not observe the mechanism mapping individual choices to market outcomes. This is natural when dealing with complex and informal processes, such as group formation. Moreover, our approach identifies the group formation mechanism itself in addition to the individuals' preference structure, which is necessary for conducting counterfactual simulations.

Broadly speaking, joint venture formation, the focus of this study, can be seen as a particular type of joint bidding, which has been studied by Cho, Jewell and Vohra (2002), Cantillon (2008), and Chatterjee, Mitra and Mukherjee (2017), including in specific contexts of procurement auctions (Gugler, Weichselbaumer and Zulehner, 2021), subcontracting (Bouckaert and Van Moer, 2021), and patent auctions (Asker, Baccara and Lee, 2021)[4][5]. The empirical literature on the joint bidding, including Iimi (2004), Estache and Iimi (2009), and Branzoli and Decarolis (2015), has developed slightly independently from the theoretical literature. Its focuse is on examining whether joint bidding has pro-competitive effects or anti-competitive effects, and generally finds the pro-competitive effect of joint bidding. A notable exception is Gugler, Weichselbaumer and Zulehner (2021), who used a structural approach, endogenizing the entry and joint venture formation in a reduced-form way with heterogeneous bidders, and finding pro-competitive effects. Our study complements Gugler, Weichselbaumer and Zulehner (2021) by explicitly modeling the joint venture formation and imposing equilibrium

[4]Recent industrial organization literature suggests that group formation, such as mergers and joint ventures, generates synergies, as reviewed by Asker and Nocke (2021). We incorporate this viewpoint into the context of joint ventures. Our model encompasses the possibility of cost synergies generated by forming a joint venture.

[5]Another strand of literature investigates joint bidding in the context of common value auctions, where auctioneers sell natural resources such as petroleum. Although the theoretical papers, including Mead (1967) and Levin (2004), have argued that joint bidding functions as a bidding ring or a collusion and hence impedes the competitive bids among bidders, empirical studies, including Millsaps and Ott (1985), Moody and Kruvant (1988), and Hendricks and Porter (1992), found pro-competitive effects of joint bidding caused by pooling information, relaxing the budget constraint and risk sharing.

restrictions, enabling us to conduct a richer set of counterfactual analyses.

Our study also contributes to the empirical works on procurement auctions. In particular, our modeling adds the joint venture formation process to procurement auction models with endogenous entry, such as those of Li and Zheng (2009) and Krasnokutskaya and Seim (2011). Their work demonstrated that decisions regarding endogenous *entry* are crucial for understanding the origins of efficiency in procurement auctions. Our study also finds that the *choice of joint venture formation* plays a decisive role in determining the efficiency of procurement auctions. Moreover, our empirical analysis focuses specifically on the scoring auction, which is categorized as multi-attribute auctions (Perrigne and Vuong, 2019). This type of auction incorporates non-monetary information to determine the winning bidder in procurement auctions and evaluates the quality of projects submitted by the bidders to aim for long-term value maximization. Recent studies by Lewis and Bajari (2011), Kong, Perrigne and Vuong (2022), and Hanazono et al. (2020) have empirically and theoretically examined this auction format.[6] We contribute to the literature by modeling and estimating the second stage as a scoring auction within a class of asymmetric auctions.

2 Background and Data

This study focuses on public procurement auctions held by five Regional Development Bureaus of the Japanese Ministry of Land, Infrastructure, Transport and Tourism (MLIT), the largest purchaser in Japan. We first provide the institutional background in Subsection 2.1, emphasizing the auction procedure and describing how we collect and construct the data. We then present descriptive statistics, in Subsection 2.2, highlighting differences between the bids from joint venture firms and those from non-joint venture firms (single bidders). Concerns about collusive behavior among Japanese construction firms, as noted by Kawai and Nakabayashi (2024), Chassang et al. (2022), Kawai et al. (2022), and Kawai and Nakabayashi (2022), prompted us to test whether this applies to our data by employing

[6] There are other auction variants worth mentioning, such as the bid-preference or set aside policies, which aim to promote equity in procurement. Notable references in this regard include Corns and Schotter (1999), Marion (2007), Krasnokutskaya and Seim (2011), Athey, Coey and Levin (2013), Nakabayashi (2013), and Rosa (2019).

the screening method proposed by Kawai and Nakabayashi (2024) in Subsection 2.3.

2.1 Institutional Background and Descriptive Statistics

This study focuses on public procurement auctions held by five Regional Development Bureaus of the MLIT. We specifically focus on these regional offices to control for auction rules. Although municipal and prefectural government offices also use procurement auctions, their auction rules do not perfectly coincide with each other. The main tasks of these regional offices include developing and maintaining public infrastructure, such as national routes, rivers, dams, and ports, under the direct control of the MLIT. Depending on these tasks, the procurement of construction work is categorized into 22 types, including civil engineering, bridge building, pavement, and landscaping.[7]

The auction format used by each Regional Development Bureau is a variant of a first price sealed-bid auction with secret reserved prices.[8] If the secret reserve price exceeds 100 million JPY in an auction, procurers are strongly encouraged to use a scoring auction, as explained in Subsection 2.3. Moreover, if the secret reserve price roughly exceeds 500 million JPY, each auction is encouraged to invite joint ventures, aiming to generate complementarities among participating firms and take advantage of scale economies. Although there are a few exceptions for large construction projects with short deadlines, the number of participating firms for each joint venture must be two or three firms in principle. Furthermore, if the secret reserve price exceeds 680 million JPY, each procurement is subject to the Agreement of Government Procurement, a multilateral agreement within the framework of the World Trade Organization (WTO), implying that equal treatment for the foreign firms must be guaranteed.[9]

Depending on their purposes, four types of joint ventures exist: (i) project-specific (Tokutei), (ii) fixed-term (Keijo), (iii) maintaining regional infrastructure-specific (Chiiki-iji), and (iv) restoration-project-specific (Fukkyu-Fukkou). In this study, we focus on the

[7]See Table 1 for the list of the categories.

[8]Secret reserved prices are unknown *ex-ante* and the bidders whose bid exceed this secret reserved prices are disqualified.

[9]These threshold values vary across municipal and prefectural governments, hence why we focus on the auctions held by each regional offices of the Japanese MLIT.

Table 1. Summary Statistics

	# of Auctions	# of JV Auctions	Rate of JV auction	Prob. JV winning	Log of Engineer's Estimate Mean	Min	Med	Max	# Bidders Mean	Min	Med	Max
Panel (A): By Reserve Prices												
Non WTO	66733	4	0.000	0.750	18.374	4.787	18.523	22.929	4.057	1	3	52
WTO	1712	245	0.143	0.408	21.025	16.097	20.885	24.350	10.411	1	10	35
Panel (B): By Construction Types												
Civil Engr.	806	120	0.149	0.475	21.284	18.081	21.161	24.350	11.428	1	10	35
PC	177	10	0.056	0.200	20.731	19.604	20.634	22.513	11.006	1	12	19
Bridge	460	97	0.211	0.381	20.919	19.524	20.807	22.782	11.311	1	12	24
machine	63	0	0.000	NaN	20.814	19.988	20.639	22.544	3.714	1	4	9
building	99	11	0.111	0.182	21.083	19.399	20.914	23.126	6.606	1	5	28
electricity	29	1	0.034	0.000	20.517	18.066	20.533	21.857	4.069	1	3	12
airconditioner	31	5	0.161	0.400	20.780	20.281	20.683	22.525	6.516	1	6	14
dredging	15	1	0.067	0.000	20.821	20.287	20.738	21.933	4.400	1	5	8
CC	7	0	0.000	NaN	20.209	19.837	20.041	20.610	9.143	4	11	14
management	18	0	0.000	NaN	17.518	16.097	17.118	20.609	2.833	1	2	8
bridgerepair	1	0	0.000	NaN	20.796	20.796	20.796	20.796	4.000	4	4	4
painting	1	0	0.000	NaN	17.824	17.824	17.824	17.824	2.000	2	2	2
slope	4	0	0.000	NaN	20.815	20.115	20.767	21.610	13.000	1	10.5	30
pavement	1	0	0.000	NaN	19.288	19.288	19.288	19.288	14.000	14	14	14
Panel (C): By Regions												
Kanto	637	130	0.204	0.462	21.181	20.102	21.044	24.350	9.722	1	9	33
Chubu	195	6	0.031	0.167	20.896	20.107	20.783	22.935	12.354	1	12	35
Hokuriku	68	13	0.191	0.462	20.955	16.519	20.906	23.598	10.529	1	11	32
Kinki	402	16	0.040	0.188	20.909	17.028	20.790	23.334	10.846	1	10	32
Chugoku	39	2	0.051	0.500	20.935	17.824	20.752	22.743	9.718	2	10	19
Shikoku	110	18	0.164	0.333	21.019	19.837	20.803	22.934	8.918	1	8.5	22
Kyushu	261	60	0.230	0.383	20.955	16.097	20.891	23.482	10.670	1	10	32
Final Sample	793	194	0.245	0.459	21.244	18.416	21.114	24.350	10.810	1	10	33

Notes; The rate of JV auctions is the number of the auctions in which at least one joint venture participates in the total number of auctions for each construction type. The second and the third panels show the details about WTO type auctions.

first type, which appears most frequently in the data.

We first collect the data from seven regional offices for all types of construction work between 2006 and 2022.[10] Procurement projects involving joint ventures are mostly observed in auctions with reserve prices above 680 million JPY, as shown in Panel (A) of Table 1. Moreover, among these 1,712 auctions, joint ventures mostly appear in two types of construction work—Civil Engineering and Bridge Building—and in five regions—Kanto, Hokuriku, Kinki, Shikoku, and Kyushu—as shown in Panels (B) and (C) in Table 1. Therefore, we focus on civil engineering or bridge building projects with reserve prices above 680 million JPY, procured in Kanto, Hokuriku, Kinki, Shikoku, and Kyushu, which leaves 793 auctions out of 66,733.

In the auctions explained above, each bid submitted by both single bidders and joint ventures is weighted by a "score," which is calculated by the combination of bidder attributes

[10]There are eight Regional Development Bureaus of MLIT, including Tohoku, Kanto, Hokuriku, Chubu, Kinki, Chugoku, Shikoku, and Kyushu. We exclude Tohoku area from our sample to remove the impact of the 2011 off the Pacific coast of Tohoku Earthquake. Note that there are several missing years for some regional development bureaus. See Appendix.

and characteristics of the procurement project, according to pre-determined scoring rules.[11] The auctioneers then compare the "effective bids," defined as the ratio of the score to the submitted bid.[12] The bidder with the highest effective bid wins the auction, implying that submitting a lower bid or obtaining a higher score increases the likelihood of winning. The minimum score is 100, whereas the maximum score varies by auction, depending on the difficulties and complexities of the projects, reaching a maximum of 200. Therefore, effective bids typically range from 0 to 200.

Figure 1 depicts the distributions of firm-level behavior in our sample: the logarithm of the effective bid, the logarithm of the bid, and the score. The blue and orange bins represent joint ventures and single bidders, respectively. The left column shows histograms of the three variables observed in all auctions, while the right column shows the bids observed in auctions where at least one joint venture exists. In Panel (a), we observe small effective bids by joint ventures, which Panel (b) shows this is due to selective entry. Panel (c) shows that joint ventures enter relatively expensive projects, even when focusing on WTO auctions. Regarding the score, both single bidders and joint ventures face similar distributions, and we do not find systematic differences in the auctions where at least one joint venture exists.

2.2 Motivating Facts

Here, we briefly examine our data to motivate the details of the structural model. We begin by observing a small number of realized JVs in the dataset. The second column of Table 1 displays the number of auctions where at least one JV exists for each construction type. The third column shows the ratio of these auctions to the total number of auctions. Even in the largest auction category, the WTO type, we find JVs in only 14.3% of the auctions. This percentage varies among construction types and regions hosting auctions. However, even in the pairing of construction types and regions where JVs are most common, the percentage is only 24.5%. When there are no obstacles to forming a joint venture, this small number appears surprising, given that JVs are more likely to win an auction once they enter, as seen

[11] For joint ventures, we observe a score for a joint venture, not for individual firms.

[12] In our data, the bids are typically in order of 10^8 JPY, implying that the effective bids become tiny values. To visibility, in the data, we multiply effective bids $\times 10^8$.

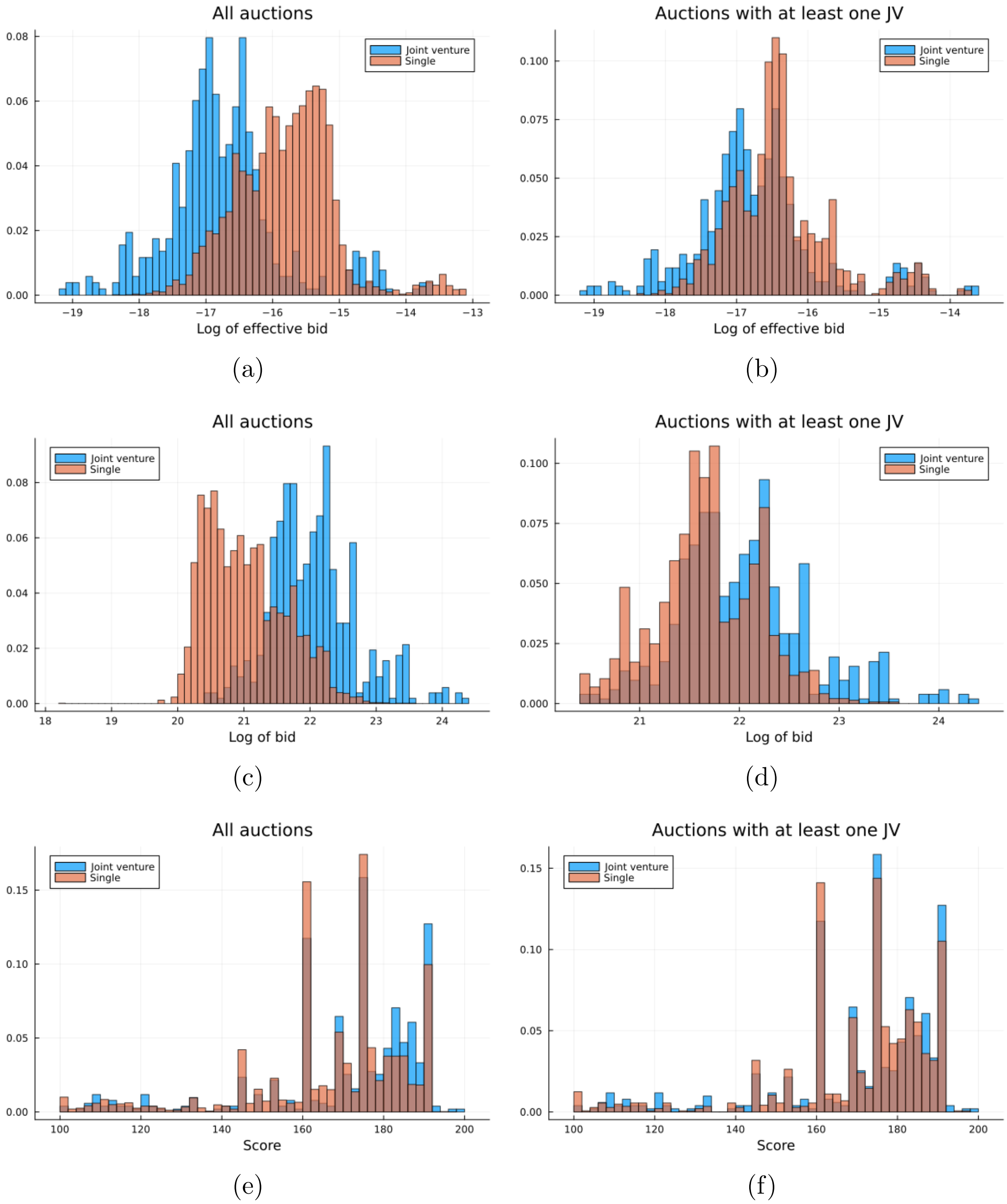


Figure 1. Histograms of effective bids, logarithm of bids, and entrants' scores.

Note: Blue bins represent joint ventures, and red bins represent single bidders. In the left column, we use all the auctions. In the right column, we use the auctions where at least one joint venture participates. The first row represents about the logarithm of effective bids, defined as the score's ratio to the bid. The second row represents about the logarithm of bids, whereas the third row displays about the score.

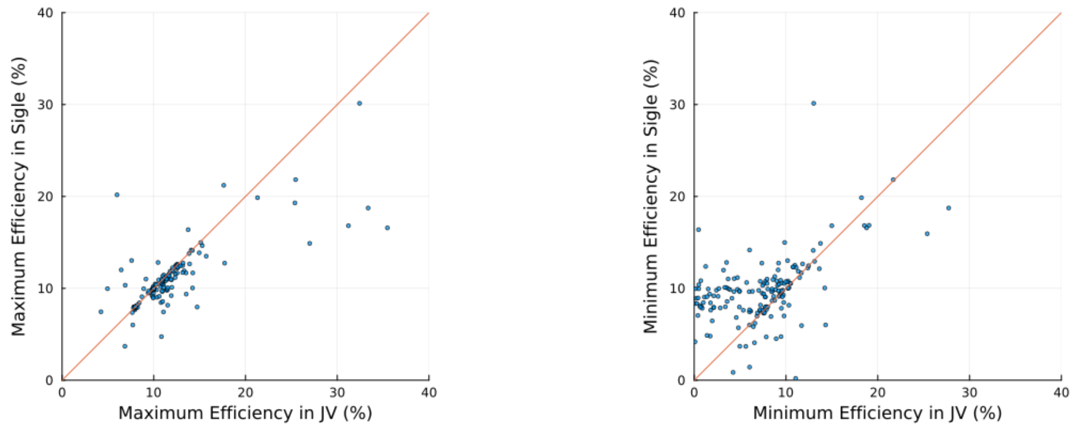
in the fourth column of Table 1. Based on this observation, we identify two obstacles in forming a joint venture: adjustment cost and search friction.

Next, we consider the nature of cost synergy. First, as shown in the fourth column of Table 1, JVs are more competitive than single bidders in auctions. While this strength undoubtedly stems from a systematic reason, various possible explanations exist. Typically, in the literature, this competitiveness arises from efficient reallocation among the firms forming a joint venture (Asker, Baccara and Lee, 2021; Bouckaert and Van Moer, 2021). In our context, this corresponds to a situation where the project cost of a JV is determined by the lowest cost among the firms forming it. In this model, JVs consistently emerge more competitive than the individual firms before forming the JV. Therefore, we expect that in an auction, JV bids are not much larger than those submitted by single bidders, even if firms with higher costs are likely to form a JV to reduce project costs.

In our data, this is not the case; JVs might bid at significantly higher prices in auctions. To compare submitted bids across auctions with varying engineer's estimates, we define a cost-efficiency measure for each bidder in each auction:

$$\text{Cost-efficiency} = \frac{\text{Engineer's Estimate} - \text{Submitted Bid}}{\text{Engineer's Estimate}} \times 100.$$

Figure 2 compares the maximum and minimum cost-efficiencies within single bidders and joint ventures. Panel (a) indicates that a JV is likely to bid at a lower price, suggesting the existence of cost synergy. However, as shown in Panel (b), the competitiveness of JVs does not always materialize: the minimum cost efficiency of JVs is relatively larger than that of single bidders. Given that the distributions of the scores are similar, this suggests that JVs might be weaker in scoring auctions. We interpret Figure 2 as evidence that (i) JV formation is not merely an efficient reallocation and (ii) the degree of cost synergy in the second stage is unknown to potential bidders in the first stage. The second point shapes the information structure in the game of auction with an entry stage. Following Levin and Smith (1994), we assume that potential entrants are unaware of their type and the type obtained when forming a JV in the auction stage prior to entry but are aware of the distributions for



(a) Maximum of efficiency (b) Minimum of efficiency

Figure 2. Comparison between single bidder and joint venture with respect to the submitted bids.

both the single bidder and the JV cases.[13] In our model, the difference between these two distributions indicates the degree of cost synergy.

As argued above, we define the degree of cost synergy as the difference between the two types of distribution at the auction stage. To clarify this concept, it is essential to define the types in the context of this study. Typically, a firm's cost information is considered private and should be included in its type. In contrast, the score is not fully known to the bidders when submitting their bids. This is because the scoring rule may include elements of uncertainty in addition to deterministic parts based on the bidders' attributes.

However, in our model, we assume that the bidders decide their bids when they know their scores. In other words, the "type" in the auction stage also includes the bidder's score. There are several explanations for this assumption. First, 15 years have passed since the Japanese government introduced these scoring rules, and it is reasonable to believe that firms can now form appropriate expectations of their scores in advance.[14] Second, as summarized

[13]This assumption regarding the information structure is reasonable and standard in the literature because a proper assessment of the project cost and score rating requires certain effort, and potential entrants must decide whether to exert such effort in the first place, which we view as the entry decision. Furthermore, for the score, potential entrants are not aware of the detailed formula in advance. All they know are the characteristics the government considers and the maximum points.

[14]Kawai and Nakabayashi (2024) provided evidence to support this information structure. They argued that some firms in the scoring auction of Japanese procurement engage in bid rotations based on the level of the score: after one firm with a high score wins an auction, another firm with a lower score takes a turn to win. Such behavior would not be observed if the firms did not know their scores in advance.

Table 2. Regression Results

	(1) ln(bid)	(2) ln(bid)	(3) ln(bid)
Score	0.003***	0.001**	0.002***
	(0.000)	(0.000)	(0.001)
# bidders		-0.021***	-0.004
		(0.001)	(0.008)
Score × # bidders			-0.000**
			(0.000)
Constant	20.537***	20.636***	20.403***
	(0.060)	(0.072)	(0.129)
Year FE		√	√
Region FE		√	√
Type FE		√	√
N	8572	8572	8572
R^2	0.010	0.172	0.173

Notes: Standard errors in parentheses. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

in Table 2, our regression analysis, in which we regress the logarithm of the submitted bids on the score and other variables, alongside some fixed effects, reveals a statistically significant positive correlation between the submitted bids and scores. This implies that the bids reflect the bidders' scores, and that such results would not occur if the bidders were unaware of their scores.

Before proceeding to our main analysis, we must examine whether the bidding patterns in our data show any indications of collusion. This step is crucial because collusive behavior is a known concern in Japanese procurement auctions, as highlighted in previous studies (e.g., Kawai and Nakabayashi, 2024; Chassang et al., 2022; Kawai et al., 2022; Kawai and Nakabayashi, 2022). If firms engage in collusion, our model and empirical strategy may not be applicable in the subsequent sections. Therefore, we use the screening method proposed by Kawai and Nakabayashi (2024) to analyze our data and confirm that it did not exhibit any suspicious bidding behaviors.

Following the approach of Kawai and Nakabayashi (2024), we first divide our data by region and year, and subsequently apply their screening method to each subset to identify any sets of auctions that may be affected by collusion.[15] The results are presented in Figures

[15]This method utilizes the mathematical principle that, when considering marginally losing and winning

A1 and A2.[16] Unlike the findings of Kawai and Nakabayashi (2024), our data does not show any suspicious bidding patterns. This discrepancy between their results and ours is likely due to the difference in the types of procurements examined. Our focus is on relatively large-scale auctions compared to those studied in the existing literature, and the bidders in our data may face much stricter compliance requirements. Therefore, we retain all the observations described in the previous subsection for our main analysis.

3 Model

We construct a model for a procurement that determines the participation decisions of potential entrants as well as the final winning price and the auction winner. Whereas our model adapts a two-stage model similar to the simplest model of auction with entry (Levin, 2004), there are two significant deviations.

First, the decision about entry is more complicated because the entry of firms can take three different modes: joint venture, single bidder and no entry, which we refer to as *entry modes*. Furthermore, the complexity arises not only from the existence of these modes but also from the fact that the joint venture entry mode cannot be achieved through an individual decision alone. Forming a joint venture requires at least one other firm to also be willing to form a joint venture, and, in addition, the firm must be able to find a suitable partner from among those interested. We call this stage by *entry and joint venture formation stage*.

Second, the two modes, JV and S, necessarily introduces the asymmetry in the auction: joint ventures and single bidders might be different in the sense of their type distribution. Moreover, the public procurement auction is conducted as a type of scoring auction. The government aims to ensure the quality of the project while also pursuing cost-effective procurement. Hence, we have to consider asymmetric scoring auction in the second stage, which will be referred to as the *bidding stage*.

bidders, the winner is randomly determined in the absence of collusion. If collusive behavior occurs, the submitted bid of the marginally losing bidder will differ unnaturally from the winning bid. For example, if a firm with a higher technology intends to lose, its submitted bids are unnaturally high. We can detect collusive bids by examining this gap. As shown in the case of the Nippo cooperation in Kawai and Nakabayashi (2024), this method can be applied to each firm, revealing a set of suspicious firms.

[16] A jump at point 0 in these figures suggests that high-quality bidders, who are likely to win carefully choose their bids to lose the auction. This is considered evidence of collusion.

Before the detailed explanation of the model, we provide an overview of the model. To begin with, the procurer (she) has a project, such as bridge construction. She must select a service provider for the project and then announces the commencement of a public procurement auction to a fixed set of potential entrants. At this moment, she also computes the *engineer's estimate* about the expected cost to complete the project but this information is not revealed to the potential entrants[17]. Our model describes the process following this announcement. In the entry and joint venture formation stage, potential entrants interact to decide their mode of entry and whether or not to participate. This interaction finally determines the number of joint ventures and single bidders in the procurement auction. We refer to these bidders as *bidders* and the pair of numbers representing the two modes as the *entry pattern*. In the bidding stage, given that the entry pattern is known among the bidders, they compete in a scoring auction to determine the winner, i.e., the service provider.

We are only concerned with the number of entrants in the two modes resulting from the entry and joint venture formation stages, not with which potential entrant forms a joint venture or enters as a single bidder. In other words, we treat all potential entrants in the procurement as homogeneous agents. This assumption of homogeneity is justified by the fact that all potential entrants must be similarly ranked in terms of firm quality before the procurement begins. While our model can accommodate heterogeneous treatment of potential entrants, doing so would come with a significant computational burden. Since this paper represents the first attempt to capture the group formation process in a fully tractable way, we focus on introducing the basic structure of the model by ignoring the heterogeneity.

3.1 Bidding stage

We start by formally describing the format of the scoring auction in this context. In a scoring auction, each bidder, denoted by i, must submit two components: a bid, b_i, and a project plan. The procurer evaluates the plan according to a predetermined scoring rule, with the score assigned to bidder i's plan denoted by s_i. In this setting, as described in Section 2,

[17]The primary purpose of keeping the engineer's estimate hidden from the market is to prevent underbidding.

bidders are evaluated based on their effective bid, $B_i \equiv \frac{s_i}{b_i}$.[18] The bidder with the highest B_i wins the auction. When the bidder i wins the auction, the payoff is given by

$$b_i - c_i = \frac{s_i}{B_i} - c_i,$$

where c_i is the individual cost for the project of the bidder i, When determining the bid, we assume that the score and cost are private information. We call these pair by *type* of a bidder.

In the following section, let M denote the number of JVs and N denote the number of single bidders. Thus, the entry pattern is represented by the tuple (M, N). Departing slightly from standard notation, we also use M and N to refer to the sets of bidders for each respective entry mode. For both entry modes, the optimal bidding strategy is determined by solving the corresponding expected payoff maximization problem. A single bidder i's problem is described as follows:

$$\max_{B} \ Pr\left(\left\{B_j^{JV} \leq B \ \forall j \in M\right\},\ \left\{B_j^{S} \leq B \ \forall j \in N \setminus \{i\}\right\}\right)\left(\frac{s_i}{B} - c_i\right), \tag{1}$$

where $\{B_j^S\}_{j \in N \setminus \{i\}}$ denotes the effective bids of other single bidders and $\{B_j^{JV}\}_{j \in M}$ denotes the effective bids of joint ventures. The objective function represents the expected payoff, where the expectation is taken with respect to the realization of the scores and costs of other bidders. The problem for a joint venture is defined similarly, replacing M with $M \setminus \{i\}$ and $N \setminus \{i\}$ with N.

The asymmetry of this auction is due to the difference of the distribution of type by entry modes. The distributions are denoted by G^S for single bidder and G^{JV} for joint venture. Both are two-dimensional distributions encompassing the scores and costs. As in the standard auction model, we assume that these types are drawn independently.

Assumption 1. *For single bidders, (s_i, c_i), is drawn independently from G^S. For joint ventures, (s_i, c_i) is drawn independently from G^{JV}.*

To determine the optimal bidding strategy in this auction, we directly consider the

[18] Different types of scoring auctions combine b_i and s_i in various ways when evaluating bidder i. Another commonly used type relies on subtraction: the uni-dimensional evaluation of the plan is computed as $s_i - b_i$.

distributions of the effective bids of the other bidders. Let F^{JV} be the distribution of the logarithm of the JV's effective bids and F^S be the same for single bidders. Then, under Assumption 1, the expected payoff maximization problem for a single bidder i, which is written in (1), can be written as follows:

$$\max_B \; F^{JV}(\ln B)^M F^S(\ln B)^{N-1}\left(\frac{s_i}{B} - c_i\right). \tag{2}$$

We also have the corresponding problem for a joint venture, denoted by j. The combined system of the first-order conditions for both entry modes is described as follows:

$$\begin{aligned} 1 - \frac{c_i}{s_i}B &= \frac{1}{(N-1)\frac{f^S(\ln B)}{F^S(\ln B)} + M\frac{f^{JV}(\ln B)}{F^{JV}(\ln B)}}, \\ 1 - \frac{c_j}{s_j}B &= \frac{1}{N\frac{f^S(\ln B)}{F^S(\ln B)} + (M-1)\frac{f^{JV}(\ln B)}{F^{JV}(\ln B)}}. \end{aligned} \tag{3}$$

This system indicates that the two dimensional type affects the effective bid only through their fraction, $\frac{c}{s}$. Hereafter we call this fraction by *effective cost*. Although our problem contains two-dimensional type, the optimization problem is reduced to a one-dimensional problem with respect to effective cost. Hence, it is easy to solve for these two first-order conditions and determine the optimal bidding strategies given the distribtuion of the other bidders' effective bids: $B_S\left(\cdot\, ; F^S, F^{JV}\right)$ for singke bidders and $B_{JV}\left(\cdot\, ; F^S, F^{JV}\right)$ for joint ventures[19]. These functions map $\frac{c}{s}$ to an effective bid depending on the entry mode.

Our equilibrium is a version of the standard Bayesian Nash equilibrium considered in the auction literature. Because we have two entry modes, our equilibrium comprises a pair of bidding strategies.

Definition 1. *The pair of two invertible functions mapping $\frac{c}{s}$ to an effective bid, $B_S^\star(\cdot)$ and $B_{JV}^\star(\cdot)$, is an equilibrium, if it satisfies*

$$\begin{cases} B_S^\star(\cdot) = B_S\left(\cdot\, ; F^S, F^{JV}\right) \\ B_{JV}^\star(\cdot) = B_{JV}\left(\cdot\, ; F^S, F^{JV}\right) \end{cases} \tag{4}$$

[19]We focus on symmetric equilibirum in the sense that all the single bidders use the same bidding function and all the joint ventures use the same bidding function.

where

$$\begin{cases} F^S(\ln B) \equiv Pr\left(B_S^\star\left(\frac{c}{s}\right) \leq B\right) \\ F^{JV}(\ln B) \equiv Pr\left(B_{JV}^\star\left(\frac{c}{s}\right) \leq B\right). \end{cases}$$

The probability is computed based on G^S for a single bidder and G^{JV} for a joint venture.

The existence and the uniqueness of this equilibrium is discussed in Hanazono et al. (2020) and Lebrun (1999). Under mild regularity assumptions, the current asymmetric scoring auction always has a unique equilibrium. For completeness, we describe the basic existence and uniqueness argument in our case in Appendix B.

Theorem 1. *Under Assumption 1, there exists the unique pair of $(B_S^\star(\cdot), B_{JV}^\star(\cdot))$ satisfying the equilibrium condition.*

We denote the maximized expected payoff for both entry modes under the unique equilibrium by $V^S(\frac{c}{s}; M, N)$ for a single bidder and $V^{JV}(\frac{c}{s}; M, N)$ for a joint venture where the effective cost is $\frac{c}{s}$. Lastly, for later use, we define the ex-ante expected payoff functions for all the entry modes. Hereafter, we use $\delta \in \{JV, S, O\}$ to represent the entry mode of a potential entrant. These values take maps an entry patterns to a expected payoff depending on entry modes.

Definition 2. *(Ex-ante expected payoff function) Ex-ante expected payoff functions are denoted by $u(M, N, \delta)$ and defined as follows:*

$$\begin{cases} u(M, N, JV) & \equiv \mathbb{E}\left[V^{JV}(\frac{c}{s}; M, N)\right], \\ u(M, N, S) & \equiv \mathbb{E}\left[V^{S}(\frac{c}{s}; M, N)\right], \\ u(M, N, O) & \equiv 0. \end{cases}$$

We will refer to the triplet (M, N, δ) as an *outcome*. Let $\bar{N}$ represent the number of potential entrants, which is finite and common knowledge. Now the number of possible entry patterns is computed as $K \equiv \bar{N} + \lfloor \frac{\bar{N}}{2} \rfloor \left(\bar{N} - \lfloor \frac{\bar{N}}{2} \rfloor\right)$. And we have three entry modes. Hence the number of possible outcomes is also finite, $3K$. Then we define a ex-ante expected payoff vector as a stack of the ex-ante expected payoff functions.

Definition 3. *(Ex-ante expected payoff vector) Ex-ante expected payoff vector is the stack of the Ex-ante expected payoff functions:*

$$u \equiv \begin{pmatrix} u(JV) \\ u(S) \\ u(O) \end{pmatrix}$$

where

$$u(\delta) = \begin{pmatrix} u(0, 0, \delta) \\ \vdots \\ u(\bar{M}, \bar{N} - 2\bar{M}, \delta) \end{pmatrix}$$

3.2 Entry and joint venture formation stage

In this section, we describe how to specify the equilibrium distribution of entry patterns in a procurement auction. In contrast to the standard entry game (Seim, 2006), the entry mode is not fully determined by the individual decision of the potential entrants. This deviation prohibits us from describing the equilibrium distribution over the entry patterns as the product of the individual distribution over entry modes. Instead, we must *directly* specify the equilibrium distribution over the entry patterns.

3.2.1 Intention choice problem

For the above purpose, we begin by describing the individual choice problem, which concerns not entry modes but rather *entry intentions*. A potential entrant initially selects an entry intention, denoted by I, from three options: JV, S, and O. JV signifies the intent to form a joint venture with another potential entrant, S indicates the intent to bid as a single entity, and O represents the decision to withdraw from the procurement process. It is natural that both S and O lead directly to an entry mode: selecting S results in participating as a single bidder in the bidding stage, and choosing O means not entering the bidding stage at all. However, choosing JV does not guarantee the formation of a joint venture, as this requires at least one other entrant to agree to the partnership. We refer to this decision over entry intentions as the *intention choice problem*.

From an econometrician's perspective, the notable aspect of the intention choice problem is that the intention itself cannot be observed. For instance, imagine observing a joint venture and a single bidder in a procurement auction. It is natural to think that the firms forming the joint venture chose JV as their intentions. However, we do not know the intention of the single bidder. The bidder might have chosen JV as their intention, failed to find a partner, and subsequently entered as a single bidder.

Now we formalize the intention choice problem of a potential entrant. The payoff in this problem is determined by the ex-ante expected payoff in the bidding stage, $u(M, N, \delta)$ as defined in Definition 2, and the entry costs for the two entry modes. We use $c(S)$ as the entry cost for a single bidder and $c(JV)$ as the entry cost for a joint venture. The cost for a joint venture includes the management cost necessary to decide the bid in the subsequent auction as a joint venture, as well as any frictional costs associated with forming the joint venture. Note that these costs are incurred when the entry mode is determined, not when the entry intention is chosen.

Hence, from the viewpoint of the potential entrant, what matters is the distribution over the outcome to compute the expected payoff attached to one intention. We use $P^I \in \mathbb{R}^{3K}$ as the probability vector over possible outcomes when the potential entrant chooses intention I, which we call *prediction vector for intention I*. This vector is composed of three parts for every entry modes: $P^I = \left(P^{I\prime}_{JV}, P^{I\prime}_{S}, P^{I\prime}_{O}\right)'$ where $P^I_\delta \in \mathbb{R}^K$ is the vector covering all the outcomes given one entry mode δ. And we use $P \equiv \left(P^{JV\prime}, P^{S\prime}, P^{O\prime}\right)'$ as a stack of the three prediction vectors. Then, the expected payoff attached with the intention I is simply defined as the product of the prediction vector for intention I and the ex-ante expected payoff vector subtracted with the corresponding entry costs.

Definition 4. *(Expected payoff attached with an intention) Expected payoff attached with intention I, which is denoted by $v(I)$, is defined as follows: for $I \in \{JV, S, O\}$,*

$$v(I) = P^I_{JV} \cdot (u(JV) - c(JV)) + P^I_S \cdot (u(S) - c(S)) + P^I_O \cdot 0.$$

We incorporate additive disturbance terms $\epsilon = (\epsilon(JV), \epsilon(S), \epsilon(O))$ to account for unobserved disturbances in choice of intention[20]. We assume that ϵ independently follows a Type I extreme value distribution for all the potential entrants. Consequently, the choice probabilities are computed as the usual Logit form: we denote the choice probabilities by $m(I)$ for $I \in \{JV, S, O\}$. We use m as the stacked vector of the three probabilities and use $m(P)$ when we emphasize it depends on the prediction vectors P.

When we focus on a potential entrant, we denote the number of the other $\bar{N} - 1$ potential entrants expressing each intention by L_1, L_2, and L_3, and refer to the triplet (L_1, L_2, L_3) as the *intention pattern of the others*. The distribution over intention pattern of the others is just a multinomial distribution with the probabilities of each intention is specified by $m(I)$. This is because the intention choice problem is just a individual discrete choice problem given P. We denote this distribution by Q and when we emphasize it depends on m and P, we use $Q(m(P))$.

With the introduction of the intention choice problem, our task now is to describe how to link the distribution of the intention patterns of the others to the distribution of entry patterns under an appropriate equilibrium concept. As a first step, we consider mapping a distribution of the intention patterns of the others, Q, to a stacked prediction vector P. Rather than detailed structural modeling, we treat the mapping from Q to P as a parameter and estimate it directly, similar to a semi-parametric model[21]. This approach helps us to avoid potential misspecifications that might arise from the detailed modeling of the joint venture formation process. For this purpose, we introduce an *outcome mixing matrix* that maps a distribution over intention patterns of the others to a stacked prediction vectors.

[20]One major source of this disturbance is the unobserved heterogeneity in the entry costs, $c(JV)$ and $c(S)$.

[21]For instance, we might consider a coalition formation model, as discussed byUetake and Watanabe (2020). Studies on coalition formation, such as the survey of Ray and Vohra (2015), have proposed numerous models for group formation. These models can analyze group formation even under externalities, as in the current situation. However, the outcomes of these models heavily depend on assumptions about players' farsightedness, as illustrated in example 5.5 in Ray and Vohra (2015), and on the protocol specifying the group formation process, like the order of proposers. For empirical work on group formation in procurement auctions, a model that avoids these specific features is needed, as the exact process of JV formation is unknown. This is why models from coalition formation literature are not adopted here.

3.2.2 Outcome mixing matrix

We introduce some notations. Since we fix one intention chosen by a potential agent among $\bar{N}$, we use the number of the possible intention patterns of the others, which is denoted by $J \equiv {}_{\bar{N}+1}C_2$. Given an intention pattern (L_1, L_2, L_3) of the others and an intention $I \in \{JV, S, O\}$, let $r(M, N, \delta \mid L_1, L_2, L_3; I)$ denote the probability that the outcome (M, N, δ) materializes. For example, $r(M, N, JV \mid L_1, L_2, L_3; JV)$ is the probability that a potential entrant choosing JV as its intention is assigned to the event that M joint ventures and N single bidders bid in the bidding stage and is able to form a joint venture when the intention pattern is (L_1, L_2, L_3).

We stack r to make a large matrix to map a distribution over the intention patterns of the others to a distribution over the entry patterns. First, for any intention $I \in \{JV, S, O\}$, we consider an *outcome mixing matrix for intention I*, which is a column-stochastic matrix.

Definition 5. *For $I \in \{JV, S, O\}$, an outcome mixing matrix for intention I is $R^I \in \mathbb{R}^{3K \times J}$ such that*

$$R^I = \begin{pmatrix} \tilde{R}^I_{JV} \\ \tilde{R}^I_S \\ \tilde{R}^I_O \end{pmatrix}$$

where $\tilde{R}^I_\delta \in \mathbb{R}^{K \times J}$ is constructed as follows:

$$\tilde{R}^I_\delta = \begin{pmatrix} r(0, 1, \delta \mid \bar{N}-1, 0, 0, I) & \dots & r(0, 1, \delta \mid 0, 0, \bar{N}-1, I) \\ \vdots & \vdots & \vdots \\ r(\bar{M}, \bar{N}-2\bar{M}, \delta \mid \bar{N}-1, 0, 0, I) & \dots & r(\bar{M}, \bar{N}-2\bar{M}, \delta \mid 0, 0, \bar{N}-1, I) \end{pmatrix}.$$

In other words, each row of $\tilde{R}^I_\delta$ contains the probabilities r for all possible intention patterns of the others corresponding to the same entry pattern. The matrix $\tilde{R}^I_\delta$ is formed by stacking these row vectors.

Without further specifications, R^I represents a high-dimensional object. To define search friction clearly, we impose a specific structure on R^I. We further parameterize R^I using two components: (Φ, ρ), where $\Phi : \{0, 1, \cdots, \bar{N}\} \to \{0, 1, \cdots, \bar{M}\}$ maps the number of potential entrants who choose JV as their intention to the number of successfully formed joint ventures, and $\rho :\in [0, 1]$ represents the probability of entering as a single bidder after a potential entrant, who has chosen JV as its intention, fails to form a joint venture. Then, we can calculate the number of potential but unformed joint ventures as $\lfloor \frac{L_1}{2} \rfloor - \Phi(L_1)$ for every feasible L_1. By examining this discrepancy, we can quantify the severity of search friction. Note that, in this formulation, the distribution of the number of single bidders, conditional on the number of JVs, becomes a mixture of multinomial distributions.

We specify the structure of Φ. First, it should be a monotone non-decreasing function: when more potential entrants try to form a joint venture, we do not expect less joint ventures are actually formed. Second, the number of formed joint ventures must increase by one. We do not allow 2 more joint ventures are formed at once.

Assumption 2. *Φ is non-decreasing function and $\Phi(L_1) - \Phi(L_1 - 1) \leq 1$ for all $L_1 \in \{1, \cdots, \bar{N}\}$.*

Moreover, as we have mentioned, we assume that the intentions of S and O directly determine the entry mode. This assumption also dramatically simplifies the structure of R^I.

Assumption 3. *$r(M, N, JV \mid L_1, L_2, L_3; S) = r(M, N, O \mid L_1, L_2, L_3; S) = r(M, N, JV \mid L_1, L_2, L_3; O) = r(M, N, S \mid L_1, L_2, L_3; O) = 0$ for all pairs of (M, N), and $\sum_{(M,N)} r(M, N, S \mid L_1, L_2, L_3; S) = 1$ for all (L_1, L_2, L_3) with $L_2 \geq 1$*

Example 1. *We present a small example of an outcome mixing matrix for intention I. There are three potential entrants. A component of an outcome mixing matrix for JV, R^{JV}_{JV}, can be represented as in Table 3. The first row represents the corresponding intention patterns of the others and the first column list the corresponding entry patterns. In this example, we require two or three potential entrants choosing JV as their intention to form one joint venture.*

Based on the three outcome mixing matrices for every intentions, R^{JV}, R^S and R^O, we construct an *outcome mixing matrix* by stacking them.

Table 3. A component of an outcome mixing matrix for intention JV: $\tilde{R}^{JV}_{JV}$

	(2,0,0)	(1,1,0)	(1,0,1)	(0,2,0)	(0,1,1)	(0,0,2)
(0,0)	0	0	0	0	0	0
(0,1)	0	0	0	0	0	0
(0,2)	0	0	0	0	0	0
(0,3)	0	0	0	1	0	0
(1,0)	$1-\rho$	0	1	0	0	0
(1,1)	ρ	1	0	0	0	0

Definition 6. *An outcome mixing matrix $R \in \mathbb{R}^{9K \times J}$ is the stack of R^i:*

$$R \equiv \begin{pmatrix} R^{JV} \\ \\ R^{S} \\ \\ R^{O} \end{pmatrix}$$

.

An outcome mixing matrix R maps a distribution over the intention patterns of the others, Q, to a stacked prediction vector, P. This is because the three components of R, R^{JV}, R^S, and R^O, are column-stochastic matrices. In short, they are connected in the following expression: $P = RQ$.

Remember that our task is to link Q to the distribution of the entry patterns. Now we denote the probability vector representing the distribution over the entry patterns by $W \in \mathbb{R}^K$. Then, under rational expectation assumption, we can decompose W as the weighted average of the prediction vectors for the three intentions.

Assumption 4.

$$W(M,N) = \sum_{I \in \{JV,S,O\}} m(I) \sum_{\delta \in \{JV,S,O\}} P^I_\delta.$$

In short, we connect the distribution of the intention pattern of the others, Q, to the distribution of the entry patterns, W, using an outcome mixing matrix, R, and the rational expectation assumption 4.

3.2.3 Equilibrium

Finally, we define the Bayesian Nash equilibrium in the entry and joint venture formation stage in order to characterize the distribution over the entry patterns. Remember that Q depends on P only through $m(P)$, and we use $Q(m(P))$ to emphasize this dependence. Then, from the above discussion, the stack of the prediction vector, P, must satisfy the following system

$$P = RQ(m(P)). \tag{5}$$

We denote the solution of this system by $P^\star$. Following the definition of the Bayesian Nash equilibrium in Seim (2006), this fixed point $P^\star$ is the stack of equilibrium prediction vectors. Based on this fixed point, we can compute the corresponding distribution of the entry patterns.

Definition 7. *(Equilibrium) $P^\star$ is the stack of equilibrium prediction vectors if $P^\star$ is the solution of the following system: $P = RQ(m(P))$. Under Assumption 4, the equilibrium distribution over the entry patterns, which is denoted by $W^\star$, is computed as follows:*

$$W^\star \equiv \sum_{I \in \{JV,S,O\}} m(I; P^\star) \sum_{\delta \in \{JV,S,O\}} P_\delta^{I\star},$$

$m(I;P)$ is the choice probability of intention I under the stack of equilibrium prediction vectors P.

For the system, $P = RQ(m(P))$, we have a unique equilibrium. This is because $RQ(m(P))$ is a contraction mapping with respect to P. This uniqueness is required for the following estimation procedure.

Theorem 2. *$P = RQ(m(P))$ has a unique fixed point. In other words, the entry and joint venture formation stage has a unique Bayesian Nash equilibrium.*

Proof. See Appendix B. □

As mentioned at the beginning of this section, it is conceptually straightforward to allow for the heterogeneous treatment of potential entrants. In such a model, the intention patterns of the others and entry patterns would need to be defined based on the combinations of each potential entrant's individual intentions and the resulting entry mode. While this formulation can be applied to the heterogeneous case, it clearly leads to a significant increase in the number of combinations, making the computation much more demanding.

4 Empirical strategy and identification

In this section, we outline the empirical strategies for the two stages separately. In the bidding stage, our aim is to estimate the distributions types for both entry modes. Once we have estimated these distributions, we proceed to calculate the ex-ante expected payoff vector, u, defined in Definition 3. In the entry and joint venture formation stage, we use the estimated ex-ante expected payoff vector to estimate two key parameters: the adjustment cost associated with managing a joint venture and the two components of the outcome mixing matrix, Φ and ρ, as described in Section 3.2.

4.1 Empirical strategy for bidding sage

We denote each public procurement auction by j. For every auction j, we observe the following elements: the set of bidders, denoted by $\mathcal{B}_j$, where i represents each bidder. We also observe the entry mode of bidder i, which could be either as a joint venture (JV) or as a single bidder. Based on this, we calculate the number of joint ventures M_j and the number of single bidders N_j. For each bidder i, we observe the submitted bids b_i and the corresponding scores s_i, which allow us to compute the effective bid B_i for all bidders. Finally, as explained in Section 2.1, the procurer calculates the engineer's estimate as the expected cost of the project for auction j, denoted by p_J.

Our estimation target is the type distributions for both entry modes, JV and single bidders. Since it is natural for the marginal distribution of the cost to depend on the engineer's estimate, we denote the two type distributions by $G_{JV}(p_j)$ and $G_S(p_j)$. We now specify their dependence on the engineer's estimate. First, we introduce a common two-dimensional distribution function, $\tilde{G}_\delta$, for the two entry modes $\delta \in \{JV, S\}$. The random variable following these distributions is denoted by $(S, \tilde{C})$. We then assume that $(S, \tilde{C}p_j)$

follows G_δ for both entry modes. We refer to $\tilde{C}$ and its realization as the *individual cost factor*. This assumption implies that the pair consisting of the score and the individual cost factor follows the same distribution, regardless of the engineer's estimate of the projects[22].

Assumption 5. *For the two entry modes $\delta \in \{JV, S\}$, there exists a unique two-dimensional distribution $\tilde{G}_\delta$ such that $(S, \tilde{C}p_j)$ follows the distribution G_δ when $(S, \tilde{C})$ follows $\tilde{G}_\delta$.*

As discussed in Section 3.1, the optimal bidding strategy is a function of the effective cost, $\frac{c}{s}$, which for procurement auction j is equal to $\frac{\tilde{c}p_j}{s}$ under Assumption 5. We denote the distribution of the ratio of the individual cost factor to the score, $\frac{\tilde{c}}{s}$, by $\tilde{H}_\delta$ for the two entry modes. Therefore, our estimation target is $\tilde{H}_{JV}$ and $\tilde{H}_S$.

From equation (3), we can recover the individual cost factor for all the bidders by the observed distribution of the effective bids. When i is a single bidder, the inversion is

$$\tilde{c}_i = \frac{s_i}{p_j B_i}\left(1 - \frac{1}{(N_j - 1)\frac{f^S(\ln B_i)}{F^S(\ln B_i)} + M_j \frac{f^{JV}(\ln B_i)}{F^{JV}(\ln B_i)}}\right), \tag{6}$$

and, when i is a JV, the inversion is

$$\tilde{c}_i = \frac{s_i}{p_j B_i}\left(1 - \frac{1}{N_j \frac{f^S(\ln B_i)}{F^S(\ln B_i)} + (M_j - 1)\frac{f^{JV}(\ln B_i)}{F^{JV}(\ln B_i)}}\right). \tag{7}$$

Hence, our remaining task is to estimate the distributions of effective bids for the two entry modes.

We describe the estimation procedure for the distribution of effective bids. First, we assume that the logarithm of effective bids of every bidders follow a normal distribution. The mean of this distribution is modeled as the polynomial function of the combination of (p, M, N) and the indicator of joint venture. This is because the optimization problem implies that the optimal level of an effective bid depends on the combination of (p, M, N) and whether the firm is a single bidder or a JV. As shown in Appendix XXX, this parametric model fits well the data.

[22]This formulation is similar to the model that includes an unobserved auction-specific cost (Krasnokutskaya and Seim, 2011). In our case, we do not include such an unobserved cost. We just include the engineer's estimate as the directly observed auction-specific cost.

Assumption 6.

$$\ln B_i \sim N\left(\theta_{ij}, \sigma_{ij}^2\right)$$

where

$$\theta_{ij} = poly(d_i, p_j, M_j, N_j; \beta)$$

where $d_i = 1$ *when* $\delta_i = JV$ *and* β *is the parameter of the polynomial function for some degree.*

We estimate β by regressing the observed effective bids on the polynomials, which recovers the means as $poly(d_i, p_j, M_j, N_j; \hat{\beta})$ where the estimated coefficients is denoted by $\hat{\beta}$. In the *heteroskedastic case*, the estimated variances of the effective bids in auction j are as follows:

$$\hat{\sigma}_{ij}^2 = \begin{cases} \frac{1}{M_j}\sum_{i':d_{i'}=1}\left(\ln B_{i'} - poly(d_{i'}, p_j, M_j, N_j; \hat{\beta})\right)^2 & \text{when } d_i = 1 \\ \frac{1}{N_j}\sum_{i':d_{i'}=0}\left(\ln B_{i'} - poly(d_{i'}, p_j, M_j, N_j; \hat{\beta})\right)^2 & \text{otherwise.} \end{cases}$$

In contrast, in the *homoskedastic case*, we aggregate all the residuals to compute just one estimate of the variance.

4.1.1 Ex-ante expected payoffs

We first note that the ex-ante expected payoff functions are auction dependent. From Definition 2, when we include the individual cost factors, the functions are written as follows:

$$u\left(M, N, \delta\right) = \mathbb{E}\left[V^{\delta}\left(\frac{\tilde{c}p_j}{s}; M, N\right)\right].$$

The inclusion of p_j requires the function to be auction dependent. Furthermore, because the engineer's estimate is a continuous variables, the vector form is not enough to express them.

Instead, we divide the space of p into two sub-groups and compute two sets of ex-ante expected payoffs. $\bar{p}$ and $\underline{p}$ denote the maximum and the minimum of the engineer's estimate in our data. p_{cut} is the cutoff for dividing the total number of auctions into two equal-sized sets.[23] We compute two representative engineer's estimate for each group, $\tilde{p_1}, \tilde{p_2}$: these are the means of the engineer's estimates in each group. Hereafter, we refer to these groups as the *size* of the procurement auction: if the engineer's estimate for a procurement falls within $[\underline{\mathrm{p}}, p_{cut}]$, the auction is considered *small*; otherwise, it is considered *large*. The size is denoted by $k \in 1, 2$, where $k = 1$ indicates the set of small auctions and $k = 2$ indicates the set of large auctions. We treat all the auctions within the same group as homogeneous ones. And our inference target in the current procedure can be written by $u(M, N, \delta, k)$ for all the pair of M, N, δ and $k \in \{1, 2\}$, where

$$u(M, N, \delta, k) \equiv \mathbb{E}\left[V^{\delta}\left(\frac{\tilde{c}\tilde{p}_k}{s}; M, N\right)\right].$$

We explain the detail procedure to infer the ex-ante expected payoffs. Fix one entry mode, δ, and one size of auctions, k. Then, for all the possible pair of M, N, we do the following procedure:

1. Draw 2000 observed bidders with entry mode δ from auctions in group k. From the estimation of the bidding stage, we know their individual cost factors, as well as their scores.

2. For each drawn bidder i, compute the maximized expected payoff under the entry pattern (M, N) given their pair of $(\tilde{c}_i, s_i)$ and the representative engineer's estimate $\tilde{p}_k$. This is done by solving the corresponding maximization problem (2).

3. Take the average of the maximized value among the drawn bidders.

4.2 Empirical strategy for entry and joint venture formation stage

To estimate the two entry costs and outcome mixing matrices, we adopt a Mathematical Programming with Equilibrium Constraints (MPEC) approach, as discussed by (Su and

[23] This split is merely for simple execution. We can change the number of splitting and we can also rely on a kernel method to estimate the ex-ante expected payoffs as functions of the value of engineer's estimate.

Judd, 2012). The estimation involves solving the following constrained maximization problem:

$$\max_{c_{JV}, c_S, \Phi, \rho} LL(c_{JV}, c_S, \Phi, \rho) \text{ subject to (5)},$$

where the objective function is the log-likelihood function constructed by the observed counts of every entry patterns. When we denote the counts of the entry pattern of (M, N) by $T(M, N)$, the log-likelihood function is defined as follows:

$$LL(c_{JV}, c_S, \Phi, \rho) \equiv \sum_{(M,N)} T(M, N) \times \ln W^{\star}(M, N),$$

where $W^{\star}$ is the equilibrium distribution over the entry patterns which is defined in Definition 7.

The unique aspect of this estimation is that Φ is a function. However, in fact, we can identify Φ as a set of jump points. Under Assumption 2, we can define the jump points of Φ at which the number of formed joint ventures increase.

Definition 8. *For $M \leq \bar{M}$, Φ_M is the maximum number of potential entrants choosing $i = JV$ at which M JVs realize: $\Phi_M \equiv \max\{n \mid n \in \Phi^{-1}(M)\}$.*

It is obvious that the function Φ itself is identified as its set of jump points $\varphi \equiv \{\Phi_0, \cdots, \Phi_{\bar{M}}\}$ where $\Phi_0 \equiv 0$. Hence, we consider the identification and estimation of φ instead of Φ itself. Furthermore, under Assumption 2, the number of possible Φ, and of course φ, is finite because the value of Φ is bounded above by $\lfloor \frac{L_1}{2} \rfloor$.

Hence, for the estimation of Φ, our estimation approach is as follows: we solve the MPEC problem with a fixed Φ to obtain the maximized log-likelihood. By comparing the log-likelihood values obtained for different Φ's, we select the Φ that yields the largest log-likelihood as our estimate. Note that this estimation strategy is feasible because the total number of feasible Φs is finite.

Furthermore, to reduce the computation time, we limit the set of possible Φ functions. This is because, while the total number of feasible Φs is finite, it increases rapidly with the number of potential entrants.[24] We consider the following set of functions:

$$\Phi(L_1;\alpha) = \max\left(0, \lfloor \alpha \times \ln \frac{L_1}{2} \rfloor\right). \tag{8}$$

Here the parameter α determines the shape of Φ. In the estimation, we first prepare a set of α's and then solve MPEC for each possible α to determine the best set of estimates.

Lastly, we specify the number of potential entrants, $\bar{N}$, and the set of α values. From our data, we collect the names of firms that have participated in the class of procurement auctions under analysis. Among these firms, we count the ones that participated in at least 80% of the procurement auctions in this class, identifying 11 firms as frequent participants. Thus, we set $\bar{N} = 11$. Additionally, we set $\bar{M} = 4$, as 95% of the targeted procurement auctions involve at most 4 joint ventures. We then search for positive values of α that satisfy the following conditions: (1) every step of the resulting Φ is at most 1, and (2) the maximum number of JVs generated by the resulting Φ is $\bar{M}$, i.e., $\Phi(\bar{N}) = \bar{M}$. We identify seven such α values. Consequently, we solve seven MPEC problems to obtain the estimation results and compare the outcomes to determine the best estimate.

4.2.1 Identification

Here we explain how the model is identified. The set of interesting parameters is $(\Phi, (c_{JV}, c_S), \rho)$ and one endogeneous object m. The point of the identification is even when Φ is known, this model is a finite mixture because we cannot observe the actual choice of agents. So we need to handle two finite mixture structures caused by m and Φ. Note that our argument does not rely on the functional form restriction for Φ specified in (8).

We overview the identification argument. Below, we explain the steps in detail.

1. We can identify Φ from the marginal distribution of W with respect to the number of JVs.

[24] To align the model with realistic scenarios, we require at least eight potential entrants; this number allows for the possibility of observing up to four JVs in an auction, which is the maximum number recorded in our sample. In this context, there are 143 feasible Φ functions. Additionally, we set the number of potential entrants at 11 to accommodate the possibility of search friction; in this situation, the total number of feasible Φs rises to 1423.

2. Given Φ, we can identify m and ρ from W.

3. By equilibirum condition (5), the stack of equilibrium prediction vectors, P, is recovered.

4. By Hotz-Miller inversion, $v(I)$ is recovered for all I.

5. (c_{JV}, c_S) are recovered from $v(I)$ and P^I given the ex-ante expected payoff vectors obtained by the estimation result from the bidding stage.

Step 1 We denote the marginal distribution of W with respect to the number of JVs by Ω. When we observe the infinite number of auctions, we nonparametrically retrieve this marginal distribution.

Given a function Φ, our model specifies this marginal distribution in the following way. The probability that M JVs exist in an auction is the probability of $\Phi_{M-1} < L_1 \leq \Phi_M$: the number of potential entrants choosing JV as its intention is larger than the jump pint Φ_{M-1} and not larger than the next jump point Φ_M. We set $\Phi_{-1} = -1$ for later use. The intention is determined by the individual discrete choice problem. Remember that the probability of choosing intention JV is denoted by $m(JV)$. Then, the probability of this event is computed as follows: for all $M \leq \bar{M} - 1$,

$$\omega\left(m(JV), \Phi_{M-1}, \Phi_M\right) \equiv \sum_{L_1 = \Phi_{M-1}+1}^{\Phi_M} m(JV)^{L_1} (1 - m(JV))^{\bar{N} - L_1}.$$

Based on the above data and the model, we say Φ is identified when the following system with respect to $m(JV)$ and $\varphi = \{\Phi_M\}_{M \leq \bar{M}-1}$ has the unique solution:

$$\begin{aligned}
&\Omega(0) = \omega\left(m(JV), \Phi_{-1}, \Phi_0\right) \\
&\Omega(1) = \omega\left(m(JV), \Phi_0, \Phi_1\right) \\
&\vdots \\
&\Omega\left(\bar{M} - 1\right) = \omega\left(m(JV), \Phi_{\bar{M}-2}, \Phi_{\bar{M}-1}\right).
\end{aligned} \tag{9}$$

If there exists such a unique solution to the system, it implies that the set of Ω which can be generated by a Φ is distinct for every Φ. Hence we can pick corresponding Φ by observing

Ω.

Now we argue that the system (9) has the unique solution for every Ω. Fix one Ω. For any two different Φ and $\tilde{\Phi}$, we have at least one pair of decreasing span and increasing span between two sequential jump point. In other words, there exist l and l' such that

$$\Phi_l - \Phi_{l-1} < \tilde{\Phi}_l - \tilde{\Phi}_{l-1}$$
$$\tilde{\Phi}_{l'} - \tilde{\Phi}_{l'-1} < \Phi_{l'} - \Phi_{l'-1}.$$

Consider the case where Ω is generated under Φ. When we pick a wrong $\tilde{\Phi}$, for l, to meet the value of $\Omega(l)$, $m(JV)$ must decrease. Instead, for l', to meet the value of $\Omega(l')$, $m(JV)$ must increase. These two cannot happen at the same time. Hence, there is no adequate $m(JV)$ to rationalize the marginal distribution for the wrong $\tilde{\Phi}$. Hence, the system has the unique solution.

Step 2 Now we know Φ. Here we show, given Φ, we can identify the correct choice probabilities over the intentions, m, because different m leads to different distribution of the entry patterns.

When we define $\tilde{R}^I \equiv \sum_{\delta \in \{JV,S,O\}} \tilde{R}^I_\delta$, the distribution over the entry patterns is written as the mixture of the $\tilde{R}^I Q(m)$ where the weight vector is m:

$$W = \sum_{I \in \{JV,S,O\}} m(I) \tilde{R}^I Q(m). \tag{10}$$

Assume two different m's, denoted by m_1 and m_2, generate the same distribution over the entry patterns. This implies

$$0 = \sum_I \tilde{R}^I \left(m_1(I) Q(m_1) - m_2(I) Q(m_2) \right).$$

This equation is equivalent to the following:

$$\left(\tilde{R}^O;\tilde{R}^{JV};\tilde{R}^S\right)\begin{pmatrix} Q(m_1)-Q(m_2) \\ m_1(JV)Q(m_1)-m_2(JV)Q(m_2) \\ m_1(S)Q(m_1)-m_2(S)Q(m_2) \end{pmatrix} = \left(0;\tilde{R}^O;\tilde{R}^O\right)\begin{pmatrix} 0 \\ m_1(JV)Q(m_1)-m_2(JV)Q(m_2) \\ m_1(S)Q(m_1)-m_2(S)Q(m_2) \end{pmatrix}.$$

Because the span of the column vectors of $\tilde{R}^O$ is distinct from the span of the column vectors of $\tilde{R}^{JV}$ and $\tilde{R}^S$, the above equation holds only if $m_1 = m_2$. Hence, for any two different m_1 and m_2, we have different W. So by observing the distribution over the entry patterns, W, we can pick a corresponding m. Furthermore, given m, ρ is also identified by searching the value solving the system (10).

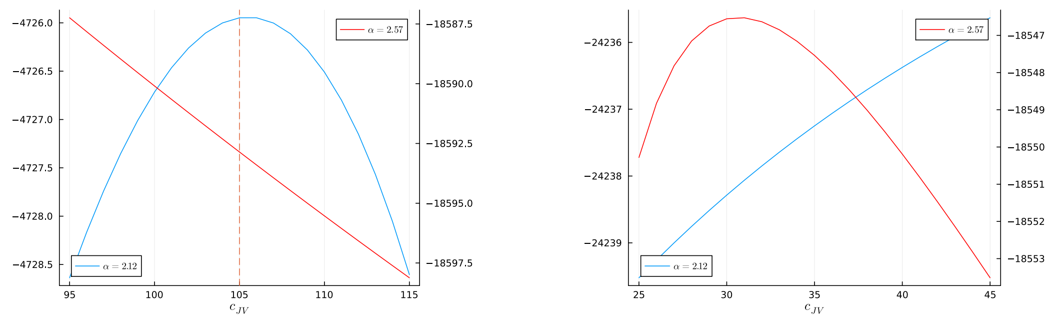
Step 3 P can be computed by the right hand side of the fixed point system: $P = RQ(m)$. This is possible because we have already obtained m, Φ and ρ.

Step 4 We have the choice probability vector over the intentions, m. Using the Hotz-Miller inversion (Hotz and Miller, 1993), we obtain the expected payoff attached with an intention, denoted by $v(I)$ for $I \in \{JV, S, O\}$, as introduced in Definition 4. In our estimation, we assume a logit error, so the inversion is simply performed by subtracting the logarithm of the choice probabilities. Remember that we normalize the utility of no entry as $v(O) = 0$.

Step 5 Recall that the expected payoffs, $v(I)$ for $I \in \{JV, S, O\}$, are constructed as outlined in 4. The left-hand side is obtained in Step 4. The prediction vectors for all the intentions are recovered in Step 3. The ex-ante expected payoff vector, u, is derived from the estimation results in the bidding stage. Thus, we have two equations—one for $I = JV$ and another for $I = S$—for the two unknowns, c_{JV} and c_S. Therefore, we can identify the cost parameters.

4.2.2 Monte Carlo simulation

We conduct a Monte Carlo simulation to demonstrate our estimation strategy and validate the identification. For visibility, we assume that c_S is already known. Imagine the case when the number of potential entrants is five and the maximum number of joint ventures is two. In this setting, we solve the equilibrium of our model under a set of ex-ante expected payoffs computed from our actual data. Based on this equilibrium distribution, we create a set of realized entries.



(a) Maximizer for the true α (b) Maximizer for the wrong α

Figure 3. The log-likelihood function of the two different α's.

Note: The blue line is obtained when we use the true α and the red line is the obtained when we use one wrong α. Panel (a) is the snapshot around true entry cost. Panel (b) is the snapshot around the maximizer of the log-likelihood of the wrong α.

Our parameter setting is as follows: $c_{JV} = 105$, $c_S = 55$, and $\alpha = 2.12$. As an alternative, we consider another possible $\alpha = 2.57$, which also yields a feasible Φ. Figure 3 summarizes the two log-likelihood functions. The left axis represents the value of the log-likelihood for the true α, and the right axis represents the same for the higher α. Panel (a) of Figure 3 displays the log-likelihood functions around the true value of c_{JV}, which is 105. We observe that when using the true α, the log-likelihood reaches its maximum around the true value. Conversely, Panel (b) of Figure 3 illustrates the region around the maximizer of the log-likelihood function for the incorrect α. The maximized log-likelihood for the true α is around -4726, while for the incorrect α, it is approximately -18547. This significant difference enables us to accurately identify the true shape of Φ.

5 Estimation results

This section presents a series of estimation results for the two-stage structural model discussed above. Section 5.1 shows the estimation results for the bidding stage: the retrieved individual costs and ex-ante expected payoff vector. Section 5.2 presents the estimation results for the entry and joint venture formation stage. Here, our main interest lies in estimating (i) Φ, which dominates the distribution of the number of successfully formed joint ventures and (ii) the two entry costs for both entry modes: c_S and c_{JV}.

5.1 Bidding stage

First, we present the estimation results for the bidding stage. Our main result is obtained when the degree of the polynomial, as specified in Assumption 6, is set to 1.[25] Figure 4 illustrates the estimate of $\tilde{H}_\delta$ for both entry modes, which represents the distribution of the ratio $\frac{\tilde{c}_i}{s_i}$. We retrieve $\tilde{c}_i$ based on the inversion discussed in Section 4.1, and compute the ratio $\frac{\tilde{c}_i}{s_i}$ for each bidder. The estimated densities are plotted using these empirical data points. In each panel, the blue line represents the density for joint ventures, and the red line represents the density for single bidders. As for the variance estimation procedure, Panel (a) corresponds to the homoskedastic case while Panel (b) corresponds to the heteroskedastic case.

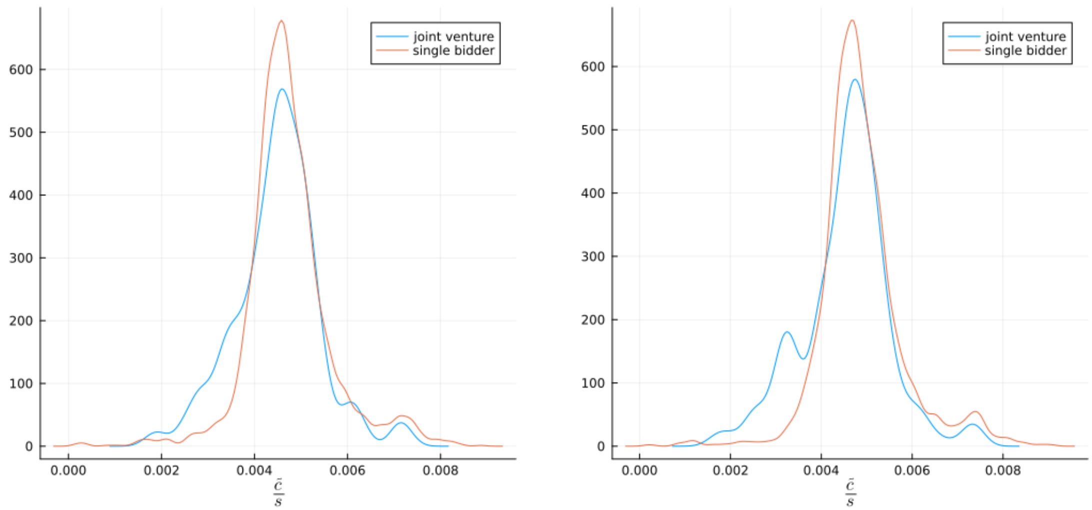
There are two observations: (i) a joint venture is likely to draw a smaller value of $\frac{\tilde{c}}{s}$, and (ii) the density of the cost-score ratio of a joint venture exhibits a bimodal distribution in Panel (b). As discussed in Section 3.1, the optimal bidding strategy is a monotone function of the fraction. Hence, this difference in distributions reflects that a joint venture is more likely to win an auction than a single bidder. Additionally, as shown in Panel (f) of Figure 1f, where the distributions of scores for single bidders and joint ventures are almost identical, it is evident that joint ventures are more competitive due to their cost-effectiveness, which we attribute to cost synergies.

The second observation suggests that two types of joint ventures exist: one that realizes cost synergies and another that does not benefit from them. The joint ventures of the latter case have a cost distribution similar to that of single bidders, indicating that the social benefits of joint ventures are primarily derived from the former type, which effectively harnesses cost synergies. Importantly, this study does not impose any specific structure on how cost synergies are materialized. Questions such as "What makes a joint venture more cost-effective?" and "How can cost efficiency be facilitated?" are topics for future studies.

Based on the estimated distribution of $(\tilde{c}_i, s_i)$, we proceed to estimate the ex-ante expected payoffs for each auction. Figure A4 displays the estimated ex-ante expected payoffs for both a single bidder and a joint venture. We categorize the auctions into two groups based on the engineer's estimate, as outlined in Section 4.1.1. For each entry pattern, we present the estimated ex-ante expected payoff. Generally, we observe that the payoffs are larger when there are fewer entrants, which is consistent with theoretical predictions.[26]

[25] We check the robustness of our results by varying the degree of the polynomial. Figure A3 in the Appendix shows the results when the degree of the polynomial is set to 2.

[26] The value for the case of a single entrant is set to the corresponding expected cost in Figure A4.



(a) A Homoskedasticity Case

(b) A Heteroskedastic Case

Figure 4. Density estimations of the distribution of the fraction $\frac{\tilde{c}}{s}$

Note: Panel (a) demonstrates the result when we assume homoskedastic variances, whereas Panel (b) demonstrates the result when we use type-dependent variance estimations. In each panel, the blue line represents the density for joint venture and red line represents the density for single bidder.

To assess the accuracy of our model, we compare the observed bids with the optimal bids derived by solving the expected profit maximization problem, as defined in Equation (2). Figure A5 compares the observed and simulated optimal effective bids for each auction size. The orange line represents the estimated density of the observed effective bids and the blue line represents the estimated density of the simulated bids. Notably, the simulated and observed bid distributions peak at similar points, although the simulated distributions exhibit thinner tails than the observed distributions.

Given that our model fits using a limited number of parameters and assumes smooth distributional characteristics, the simulated distribution inevitably has thinner tails. While a better fit might be achievable by incorporating more auction heterogeneity, such an approach would increase the computational complexity and possibly reduce the reliability of the estimates owing to outlier observations.[27] Considering this trade-off, we adopt the current specification.

[27]Examples of such methodologies include dividing the sample of auctions into three or more classes based on size, using a kernel method as discussed in Section 4.1, or considering other characteristics of the auctions.

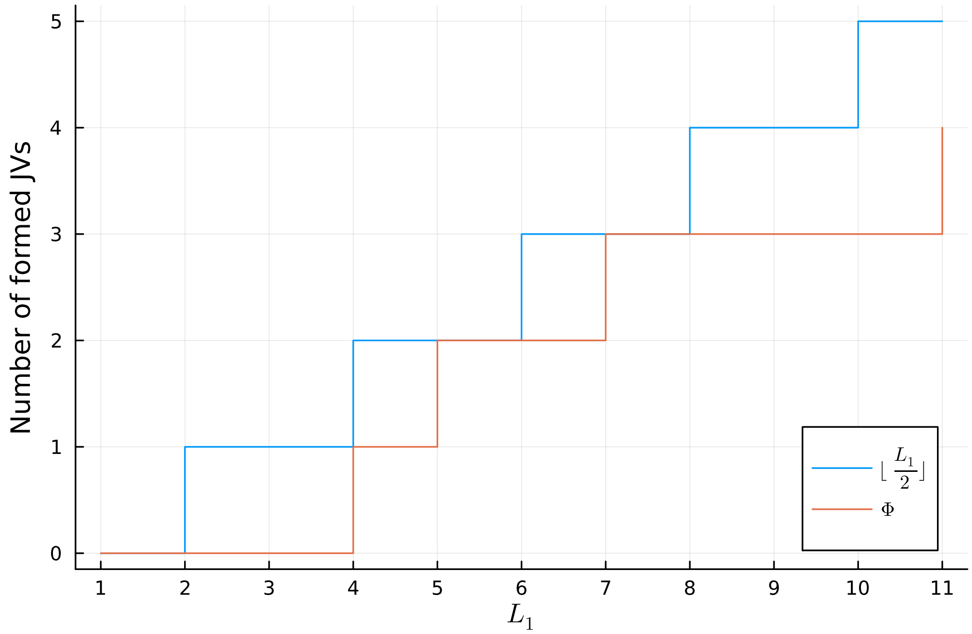


Figure 5. Estimated Number of Successfully Formed Joint Ventures for the Number of Potential Entrants Choosing JV as its Intention

Note: The horizontal axis is the total number of potential entrants who choose JV. The vertical line is the number of formed joint ventures. Blue line is the upper bound for the number of joint ventures, i.e., $\lfloor \frac{L_1}{2} \rfloor$.

5.2 Entry and joint venture formation stage

This section presents the estimation results for the entry and joint venture formation stage. Figure 5 illustrates the estimated Φ with an orange line, and the upper bound of the number of joint ventures is shown by a blue line. The horizontal axis represents the number of potential bidders choosing JV (L_1), and the vertical axis displays the number of realized joint ventures (M). The estimated Φ suggests that no joint ventures are formed when fewer than four potential entrants attempt to form a joint venture. Beyond this threshold, the number of realized joint ventures begins to increase proportionally with the number of potential entrants choosing JV up to $L_1 = 7$. When L_1 exceeds 7, we observe another flat region in the Φ function.

The discrepancy between the two lines in Figure 5 represents the search friction encountered during joint venture formation. This friction arises when potential entrants seek partners willing to form a joint venture. Various factors could contribute to this friction. For example, a potential entrant might discover significant differences in firm culture with a prospective partner, making

Table 4. Estimation Results in Entry Stage

α	2.373	2.413	2.453	2.563	2.683	2.794	2.904
c_{JV}	18.651	19.368	14.266	14.104	14.053	16.408	2.570
		(0.511)					
c_S	11.013	11.002	10.662	10.665	10.667	10.608	0.474
		(0.032)					
LL	-1963	-1955	-2065	-2062	-2054	-2039	-2722

Notes: The unit of c_{JV} and c_S is one million yen. Standard errors are computed only for α that gives the highest log-likelihood, which are contained in the brackets below the estimated values.

continued collaboration challenging after the initial meetings. Our concept of search friction encompasses all possible reasons for this deviation from the theoretically maximum number of joint ventures.

Table 4 presents the estimated entry costs for a joint venture (c_{JV}) and a single bidder (c_S) across different values of α. The α that yields the highest log-likelihood, along with the corresponding estimated entry costs, are $\alpha = 2.413$, $c_{JV} = 19.368$, and $c_S = 11.002$. The standard errors demonstrate that the difference between c_{JV} and c_S is statistically significant[28]. Entering as a joint venture necessitates additional spending of approximately 8.37 million yen by potential entrants[29]. This underlines the presence of adjustment costs in managing a joint venture, a concept widely acknowledged in the merger literature.

These adjustment costs represent approximately 0.21% of the average engineer's estimates in larger-sized auctions and 0.85% in smaller-sized auctions. Whereas these percentages might seem small, they become significant when compared to the inferred ex-ante expected payoffs, due to the low probability of winning an auction. For instance, in smaller-sized auctions with one joint venture and one single bidder, the adjustment costs constitute approximately 23.72% of the expected payoff for a joint venture. Therefore, it can be concluded that these adjustment costs may be a critical factor in explaining the relatively small number of joint ventures observed, despite their competitive advantage in the auction stage.

[28] In computation of standard errors, we fix the selected Φ and also treat the estimated ex-ante expected payoffs as fixed.

[29] This amounts to 55,760 USD at the exchange rate on October 22, 2024.

6 Counterfactual analysis

We conduct a counterfactual analysis in which we change the entry cost of joint ventures to investigate whether the government should further promote joint venture formations in procurement auctions. According to our estimation results, joint venture formation should be encouraged because of cost synergies. However, an increasing number of joint ventures may decrease the incentive to participate in an auction due to the competitiveness of joint ventures. Therefore, we must quantify the gross benefits of this policy by considering the two countervailing effects.

In principle, there are three possible policy interventions to encourage joint venture formation: (i) lowering the entry cost for joint ventures, (ii) mitigating search friction, and (iii) changing scoring rules to provide preferential treatment to joint ventures. In this study, we focus on the first intervention. We opt not to pursue the second intervention due to its complexity: since search friction in our model is expressed as the function Φ, there are numerous ways to modify Φ to bring it closer to the upper bound, making it challenging to effectively reduce search friction. Regarding the third intervention, our data lacks the detailed information necessary to compute a counterfactual score for each bidder, which inhibits us from performing counterfactual simulations of this nature.

To measure the efficiency of auction outcomes, we define *procurement efficiency* for a set of auctions as follows:

$$\frac{\sum_j (\text{Engineer's Estimates})_j - \sum_j (\text{Winning bid})_j}{\sum_j (\text{Engineer's Estimates})_j} \times 100.$$

In small-sized auctions, the observed level of procurement efficiency is 11.31, and that for large-sized auctions is 12.25. In the following counterfactual analysis, we compute this value for each auction class in a scenario where the government alters the adjustment cost.

For this counterfactual analysis, we require simulated sets of auctions and entrants. Below, we describe how they are generated based on our estimation results.

1. Set the total number of auctions, T, for a class of auction, which we divide by an engineer's estimate.

2. From the empirical distribution of the engineer's estimate within a class of auction, we draw T engineer's estimates.

3. Within a class of auction, based on the estimated values of (α, c_S, ρ) and ex-ante expected

payoffs and an arbitrarily chosen c_{JV}, compute the equilibrium prediction P^{fp} and the distribution over the entry patterns.[30]

4. Draw an entry pattern for each auction within a class based on the above equilibrium distribution.

5. Draw an individual cost factor and score for each firm from the estimated distributions for single bidders and joint ventures.

6. Compute the optimal bid by solving the optimization problem for each bidder.

7. Determine the winner in each auction and decide the winning bid by comparing the effective bids.

6.1 Simulation results

Figure 6 illustrates the simulated procurement efficiencies for each auction class. The blue solid line corresponds to the simulated procurement efficiencies for small-sized auctions, whereas the orange solid line represents those for large-sized auctions. The green dotted vertical line marks the estimated entry cost for a joint venture ($\tilde{c}_{JV}$), and the purple dotted vertical line indicates the estimated entry cost for a single bidder ($\tilde{c}_S$).

The effects of altering the adjustment cost for joint venture formation vary depending on the auction size. In small-sized auctions, an increase in the adjustment cost for a joint venture does not significantly impact procurement efficiency. This is attributed to an increase in the number of bidders resulting from the reduced formation of joint ventures. Specifically, potential entrants who might have entered as a joint venture choose to enter as single bidders instead, leading to more competitive bids. Conversely, in large-sized auctions, an increase in the adjustment cost leads to a decrease in procurement efficiency. In these auctions, the negative impact of diminished cost synergies due to fewer joint venture formation outweighs the positive effect of having more bidders.

Next, we analyze the effect of encouraging joint venture formation by decreasing adjustment costs. The area between the two dotted vertical lines in Figure 6 corresponds to this policy.

[30] We have a technical remark on this counterfactual simulation. We set the range of c_{JV} as $c_{JV} \in [7, 30]$ to compute the procurement efficiencies. In the range, we check the model can be solved, i.e., we compute the equilibrium level of predictions and the corresponding distribution over the entry patterns. Out of this range, our fixed-point problem might not behave well, which makes it difficult to compute the counterfactual distribution. Regardless, we still believe that $[7, 30]$ is enough to consider the realistic situations.

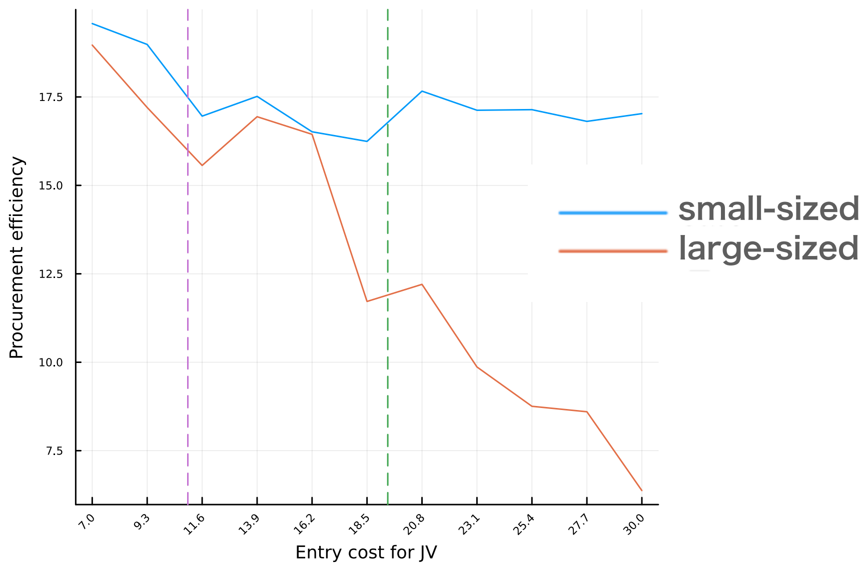


Figure 6. Counterfactual procurement efficiency vs the entry cost as a joint venture

Notes: The blue solid line represents the computed procurement efficiency in the smaller-sized auctions, whereas the orange solid line represents the same one for the larger-sized auctions. The green dotted line corresponds to the estimated value of the entry costs as joint venture, $\tilde{c}_{JV}$*, and the purple dotted line represents the estimated value of the entry costs as single bidders,* $\tilde{c}_S$*.*

Although the overall effects vary depending on the size of the auctions, particularly in larger auctions, we find that a mild reduction in the adjustment cost is key to increasing procurement efficiency. Excessive reduction leads to more joint venture entries, potentially deterring other potential entrants. Consequently, the pro-competitive effect of cost synergy is overwhelmed by the anti-competitive effect of the decreasing number of bidders. On the other hand, by slightly reducing the adjustment cost, auctioneers can benefit from the cost synergies of joint ventures, as indicated by the sharp decrease just to the left of the green dotted line.

Overall, altering the adjustment cost for a joint venture has a more significant impact on large auctions compared to small auctions. The blue line, representing small-sized auctions, remains almost constant regardless of the value of c_{JV}. This heterogeneity in responses can guide policymakers to focus on the types of auctions that are most sensitive to these changes. Additionally, policymakers should be aware of the delicate nature of implementing cost-changing policies. In large auctions, procurement efficiency is notably sensitive to small changes in c_{JV}, which is a consequence of the low expected payoffs at the auction stage.

7 Conclusion

We examine joint bidding in procurement auctions using Japanese procurement data, developing a two-stage model. In this model, a potential entrant decides on entry and joint venture formation in the first stage, and the realized bidders compete in a scoring auction in the second stage.

The estimation results from the second stage show the presence of cost synergies for joint ventures. This is apparent when comparing the estimated joint distribution of individual costs and scores for single bidders with that of joint ventures.

Our empirical findings from the first stage reveal the existence of search frictions and adjustment costs associated with joint venture formation, which hinder their formation. Furthermore, the presence of joint ventures may reduce entry incentives for single bidders, as joint ventures are often more competitive when successfully formed. Therefore, from the procurers' perspective, optimizing government procurement efficiency requires balancing the pro-competitive effects of cost synergies with the anti-competitive impacts of a reduced number of bidders. Our counterfactual simulations indicate that, depending on the auction size, overly promoting joint ventures by lowering their entry costs could worsen procurement efficiency due to a decrease in the number of participating firms.

Finally, we acknowledge the limitations of our study and outline potential directions for future research. Our model assumes homogeneity among potential entrants and focuses solely on the realized number of joint ventures and single bidders, without delving into their individual identities. This approach is driven by the empirical context of our study, where all potential bidders for World Trade Organization (WTO) type auctions are required to be pre-registered and qualified companies. Therefore, we argue that the heterogeneity among potential entrants may not be a decisive factor in evaluating the efficacy of joint ventures. Nevertheless, incorporating identity information into the analysis could allow for a more detailed examination of cost synergies, search frictions, and adjustment costs. Policymakers would benefit from understanding which specific pairs of companies are more effective in achieving the desired outcomes.[31]

Incorporating the dynamic aspects of joint venture formation is another important area for future research. For instance, the adjustment cost associated with forming a joint venture might

[31] Our estimation method accommodates such heterogeneity by expanding the range of possible entry patterns and intention patterns. Although in this paper we define the entry pattern as the combination of the number of joint ventures and the number of single bidders, it is possible to consider the power set of entry decisions made by each potential entrant, thereby accounting for their identities.

decrease over time as firms gain experience from repeated joint venture formations or even from unsuccessful attempts at forming them. This potential for a "learning-by-doing" effect could significantly impact how firms approach joint ventures and the overall dynamics within procurement auctions.

References

Agarwal, Nikhil, and Paulo Somaini. 2018. "Demand Analysis Using Strategic Reports: An Application to a School Choice Mechanism." *Econometrica*, 86(2): 391–444.

Agarwal, Nikhil, and Paulo Somaini. 2020. "Revealed Preference Analysis of School Choice Models." *Annual Review of Economics*, 12(Volume 12, 2020): 471–501.

Akkus, Oktay, J. Anthony Cookson, and Ali Hortaçsu. 2016. "The Determinants of Bank Mergers: A Revealed Preference Analysis." *Management Science*, 62(8): 2241–2258.

Asker, John, and Volker Nocke. 2021. "Collusion, mergers, and related antitrust issues." In *Handbook of Industrial Organization, Volume 5*. Vol. 5 of *Handbook of Industrial Organization*, , ed. Kate Ho, Ali Hortaçsu and Alessandro Lizzeri, 177–279. Elsevier.

Asker, John, Mariagiovanna Baccara, and SangMok Lee. 2021. "Patent Auctions and Bidding Coalitions: Structuring the Sale of Club Goods." *The RAND Journal of Economics*, 52(3): 662–690.

Athey, Susan, Dominic Coey, and Jonathan Levin. 2013. "Set-Asides and Subsidies in Auctions." *American Economic Journal: Microeconomics*, 5(1): 1–27.

Bouckaert, Jan, and Geert Van Moer. 2021. "Joint bidding and horizontal subcontracting." *International Journal of Industrial Organization*, 76: 102727.

Branzoli, Nicola, and Francesco Decarolis. 2015. "Entry and Subcontracting in Public Procurement Auctions." *Management Science*, 61(12): 2945–2962.

Cantillon, Estelle. 2008. "The effect of bidders' asymmetries on expected revenue in auctions." *Games and Economic Behavior*, 62(1): 1–25.

Chassang, Sylvain, Kei Kawai, Jun Nakabayashi, and Juan Ortner. 2022. "Robust Screens for Noncompetitive Bidding in Procurement Auctions." *Econometrica*, 90(1): 315–346.

Chatterjee, Kalyan, Manipushpak Mitra, and Conan Mukherjee. 2017. "Bidding rings: A bargaining approach." *Games and Economic Behavior*, 103: 67–82. John Nash Memorial.

Cho, In-Koo, Kevin Jewell, and Rajiv Vohra. 2002. "A Simple Model of Coalitional Bidding." *Economic Theory*, 19(3): 435–457.

Corns, Allan, and Andrew Schotter. 1999. "Can Affirmative Action Be Cost Effective? An Experimental Examination of Price-Preference Auctions." *The American Economic Review*, 89(1): 291–305.

Estache, Antonio, and Atsushi Iimi. 2009. "Joint Bidding, Governance and Public Procurement Costs: A Case of Road Projects." *Annals of Public and Cooperative Economics*, 80(3): 393–429.

Fox, Jeremy T. 2018. "Estimating matching games with transfers." *Quantitative Economics*, 9(1): 1–38.

Gowrisankaran, Gautam. 1999. "A Dynamic Model of Endogenous Horizontal Mergers." *The RAND Journal of Economics*, 30(1): 55–83.

Gugler, Klaus, Michael Weichselbaumer, and Christine Zulehner. 2021. "Evaluation of bidding groups in first-price auctions." *mimeo.*

Hanazono, Makoto, Yohsuke Hirose, Jun Nakabayashi, and Masanori Tsuruoka. 2020. "Theory, Identification, and Estimation for Scoring Auctions."

Hendricks, Kenneth, and Robert H. Porter. 1992. "Joint Bidding in Federal OCS Auctions." *The American Economic Review*, 82(2): 506–511.

Hotz, V. Joseph, and Robert A. Miller. 1993. "Conditional Choice Probabilities and the Estimation of Dynamic Models." *The Review of Economic Studies*, 60(3): 497–529.

Igami, Mitsuru, and Kosuke Uetake. 2020. "Mergers, Innovation, and Entry-Exit Dynamics: Consolidation of the Hard Disk Drive Industry, 1996–2016." *The Review of Economic Studies*, 87(6): 2672—-2702.

Iimi, Atsushi. 2004. "(Anti-)Competitive effect of joint bidding: evidence from ODA procurement auctions." *Journal of the Japanese and International Economies*, 18(3): 416–439.

Kawai, Kei, and Jun Nakabayashi. 2022. "Detecting Large-Scale Collusion in Procurement Auctions." *Journal of Political Economy*, 130(5): 1364–1411.

Kawai, Kei, and Jun Nakabayashi. 2024. "A Field Experiment on Antitrust Compliance." *mimeo.*

Kawai, Kei, Jun Nakabayashi, Juan Ortner, and Sylvain Chassang. 2022. "Using Bid Rotation and Incumbency to Detect Collusion: A Regression Discontinuity Approach." *The Review of Economic Studies*, 90(1): 376–403.

Kong, Yunmi, Isabelle Perrigne, and Quang Vuong. 2022. "Multidimensional Auctions of Contracts: An Empirical Analysis." *The American Economic Review*, 112(5): 1703–36.

Krasnokutskaya, Elena, and Katja Seim. 2011. "Bid Preference Programs and Participation in Highway Procurement Auctions." *The American Economic Review*, 101(6): 2653–86.

Lebrun, Bernard. 1999. "First Price Auctions in the Asymmetric N Bidder Case." *International Economic Review*, 40(1): 125–142.

Levin, Dan. 2004. "The Competitiveness of Joint Bidding in Multi-Unit Uniform-Price Auctions." *The RAND Journal of Economics*, 35(2): 373–385.

Levin, Dan, and James L. Smith. 1994. "Equilibrium in Auctions with Entry." *The American Economic Review*, 84(3): 585–599.

Lewis, Gregory, and Patrick Bajari. 2011. "Procurement Contracting With Time Incentives: Theory and Evidence." *The Quarterly Journal of Economics*, 126(3): 1173–1211.

Li, Tong, and Xiaoyong Zheng. 2009. "Entry and Competition Effects in First-Price Auctions: Theory and Evidence from Procurement Auctions." *The Review of Economic Studies*, 76(4): 1397–1429.

Marion, Justin. 2007. "Are bid preferences benign? The effect of small business subsidies in highway procurement auctions." *Journal of Public Economics*, 91(7): 1591–1624.

Mead, Walter J. 1967. "Natural Resource Disposal Policy - Oral Auction versus Sealed Bids." *Natural Resources Journal*, 7(2): 194–224.

Miller, Nathan H., and Matthew C. Weinberg. 2017. "Understanding the Price Effects of the MillerCoors Joint Venture." *Econometrica*, 85(6): 1763–1791.

Millsaps, Steven W., and Mack Ott. 1985. "Risk Aversion, Risk Sharing, and Joint Bidding: A Study of Outer Continental Shelf Petroleum Auctions." *Land Economics*, 61(4): 372–386.

Moody, Carlisle, and W.J. Kruvant. 1988. "Joint Bidding, Entry, and the Price of OCS Leases." *The RAND Journal of Economics*, 19(2): 276–284.

Nakabayashi, Jun. 2013. "Small business set-asides in procurement auctions: An empirical analysis." *Journal of Public Economics*, 100: 28–44.

Perrigne, Isabelle, and Quang Vuong. 2019. "Econometrics of Auctions and Nonlinear Pricing." *Annual Review of Economics*, 11(1): 27–54.

Ray, Debraj, and Rajiv Vohra. 2015. "Chapter 5 - Coalition Formation." In . Vol. 4 of *Handbook of Game Theory with Economic Applications*, , ed. H. Peyton Young and Shmuel Zamir, 239–326. Elsevier.

Rosa, Benjamin V. 2019. "Resident Bid Preference, Affiliation, and Procurement Competition: Evidence from New Mexico." *The Journal of Industrial Economics*, 67(2): 161–208.

Seim, Katja. 2006. "An Empirical Model of Firm Entry with Endogenous Product-Type Choices." *The RAND Journal of Economics*, 37(3): 619–640.

Shapiro, Carl, and Robert D. Willig. 1990. "On the Antitrust Treatment of Production Joint Ventures." *Journal of Economic Perspectives*, 4(3): 113–130.

Su, Che-Lin, and Kenneth L. Judd. 2012. "Constrained Optimization Approaches to Estimation of Structural Models." *Econometrica*, 80(5): 2213–2230.

Uetake, Kosuke, and Yasutora Watanabe. 2020. "Entry by Merger: Estimates from a Two-Sided Matching Model with Externalities." *mimeo.*

Appendix A Additional figures

Appendix A.1 Kawai-Nakabayashi test for collusion

We demonstrate a series of results for the collusion screening test developed by Kawai and Nakabayashi (2024) for each subset of region in Figure A1 and for each year in Figure A2.

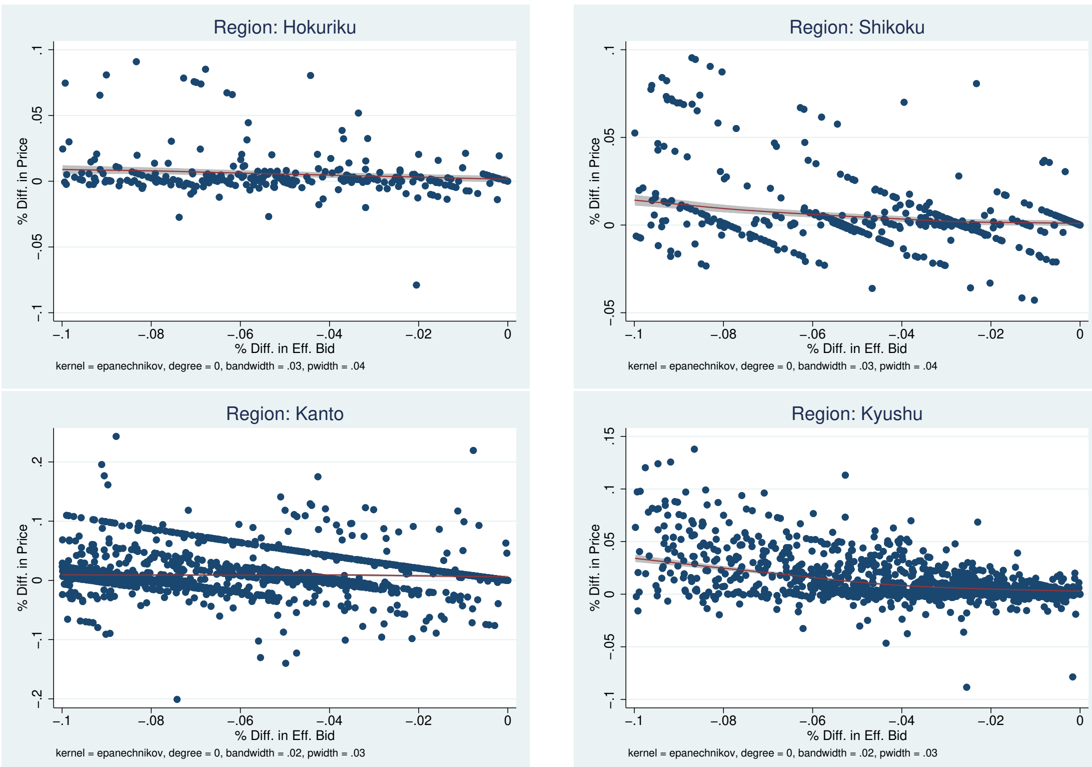


Figure A1. Region-by-Region Collusion Screening Tests by Kawai and Nakabayashi (2022)

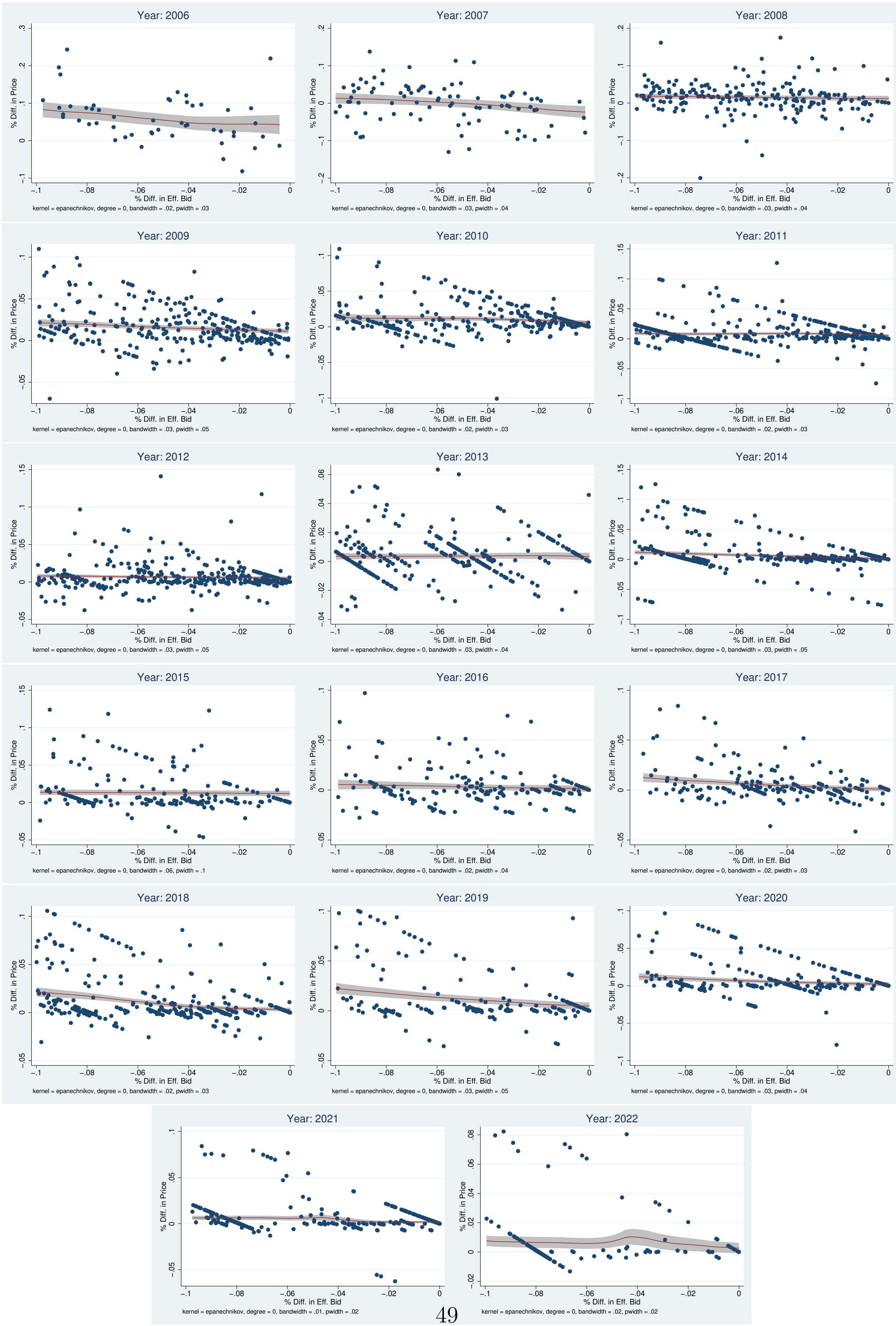
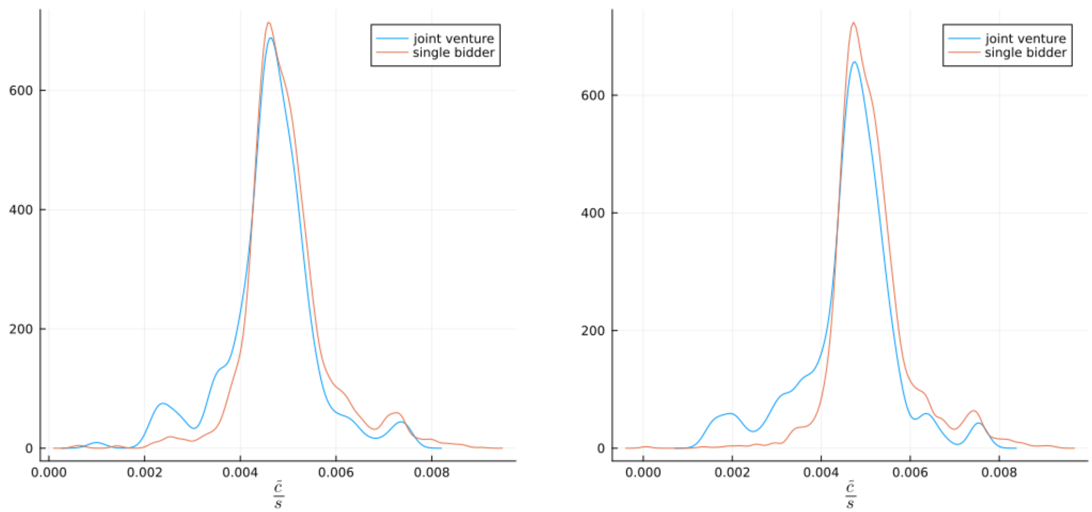


Figure A2. Year-by-Year Collusion Screening Tests by Kawai and Nakabayashi (2022)

Appendix A.2 Estimates of the density of cost-score ratio

Figure A3 shows the estimation results when we use the polynomials with degree 2. Basically, we see the same pattern as in the case of degree 1.



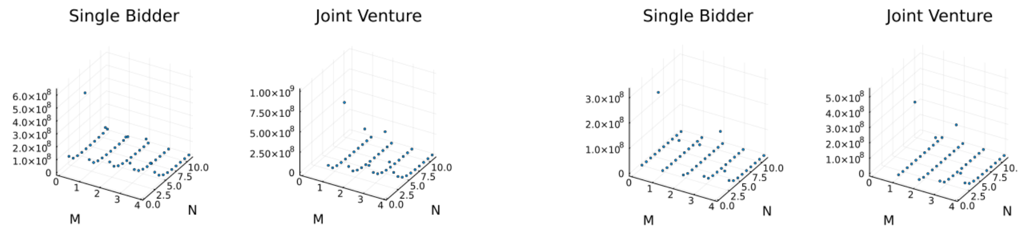
(a) Homoskedasticity case (b) Type dependent variance case

Figure A3. Density estimations of $\frac{\tilde{c}_i}{s_i}$.

Notes: We focus on the entrants in the auctions in which at least one joint venture participate. Blue line represents the density for joint venture and red line represents the density for single bidder. Panel (a) is the result when we assume homoskedastic variances. Panel (b) is the result when we use type-dependent variance estimations.

Appendix A.3 Ex-ante expected payoff

Figure A4 shows the ex-ante expected payoffs for each class of auctions. The results here are obtained when assuming the polynomial of degree 1.



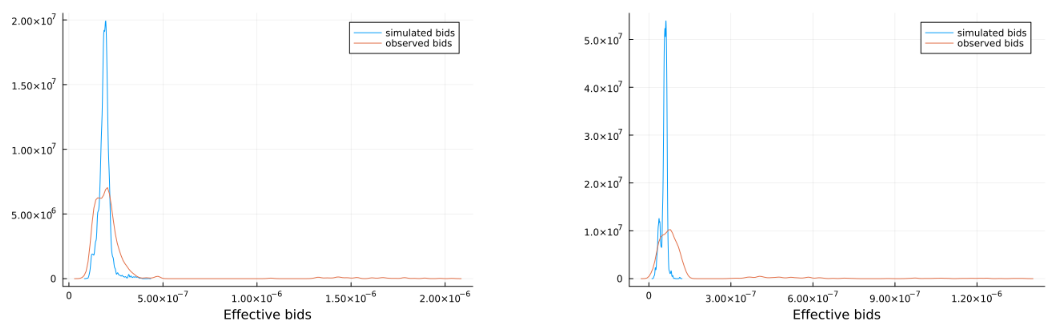
(a) Smaller size (b) Larger size

Figure A4. Estimates of the ex-ante expected payoffs.

Notes: For each category of auctions, we compute the expected payoffs by entering the auction for a single bidder and a joint venture given the number of entering single bidders and joint ventures.

Appendix A.4 Model fit

Here we compare the simulated optimal effective bids with the observed effective bids. Figure A5 shows the comparison results for all the classes of auctions.



(a) Smaller size (b) Larger size

Figure A5. Comparison of the observed bids with the computed optimal bids.

Notes: For each category of auctions, the blue line represents the estimated density function of computed optimal effective bids solving the profit maximization problem and the orange line is the estimated density of the observed effective bids.

Appendix B Proofs

Appendix B.1 Uniqueness of the equilibrium in the second stage

Here, we discuss the existence and the uniqueness of the equilibrium in this auction according to the argument of Lebrun (1999), which argues the existence and the uniqueness of the Bayesian Nash equilibrium in the first price asymmetric auction. Though our model are different from its setting in several aspects, we can still apply the similar argument to prove the existence and uniqueness of the equilibrium.

Now, assume that there is an equilibrium in this game and the bidding strategy is decreasing in $\frac{\tilde{c}_i}{s_i}$. This means that when the firm faces a higher cost or the score is lower, i.e., the firm is more competitive, the firm submits lower effective bids. If the bidding strategy is increasing in $\frac{\tilde{c}_i}{s_i}$, the reverse is true; then each entrant has an incentive to deviate to submitting more competitive bids. Hence it is not restrictive that we assume the bidding strategy is decreasing in $\frac{\tilde{c}_i}{s_i}$. Under an equilibrium, the problem (2) is transformed into the following:

$$\max_B \left(1 - \tilde{G}^S\left(B_S^{\star,-1}(B)\right)\right)^{N-1}\left(1 - \tilde{G}^{JV}\left(B_{JV}^{\star,-1}(B)\right)\right)^M\left(\frac{s_i}{B} - \tilde{c}_i p\right).$$

Here, we use the fact that $B_S^\star(\cdot)$ and $B_{JV}^\star(\cdot)$ are decreasing. We have an analogous problem for a joint venture. By taking the first order conditions for both types and solving the system of differential equations, we have the following two differential equations:

$$\begin{cases} \frac{d}{dB} B_{JV}^{\star,-1}(B) = \frac{1}{N+M-1}\frac{1-\tilde{G}^{JV}\left(B_{JV}^{\star,-1}(B)\right)}{\tilde{g}^{JV}\left(B_{JV}^{\star,-1}(B)\right)}\frac{1}{B}\frac{1}{B_{JV}^{\star,-1}(B)Bp-1}, \\ \frac{d}{dB} B_{S}^{\star,-1}(B) = \frac{1}{N+M-1}\frac{1-\tilde{G}^{S}\left(B_{S}^{\star,-1}(B)\right)}{\tilde{g}^{S}\left(B_{S}^{\star,-1}(B)\right)}\frac{1}{B}\frac{1}{B_{S}^{\star,-1}(B)Bp-1}. \end{cases}$$

Note that the right hand side is continuous in B over $[0, \frac{s_i}{\tilde{c}_i p}]$ for each i in both equations. We assume that the right hand side is Lipschitz continuous w.r.t. $B_{JV}^{\star,-1}(B)$ in the first equation and $B_S^{\star,-1}(B)$ in the second equation.

Assumption 7. $\frac{1-\tilde{G}^{JV}(\alpha)}{\tilde{g}^{JV}(\alpha)}\frac{1}{\alpha Bp-1}$ *and* $\frac{1-\tilde{G}^{S}(\alpha)}{\tilde{g}^{S}(\alpha)}\frac{1}{\alpha Bp-1}$ *are Lipschitz continuous w.r.t.* α.

Then there is a solution to the differential equation of each type and that is unique given an initial condition. This implies that the equilibrium of this auction is unique up to the initial condition if there is at least one equilibrium. Note that no firm bids the bid bigger than the cost:

for every entrant i, we have $b_i > \tilde{c}_i p \Leftrightarrow \frac{\tilde{c}_i}{s_i} p B_i - 1 < 0$. This implies that $\frac{d}{dB} B_{JV}^{\star,-1}(B)$ and $\frac{d}{dB} B_S^{\star,-1}(B)$ are negative, which does not contradict with our assumption as $B_{JV}^{\star}(\cdot)$ and $B_S^{\star}(\cdot)$ are decreasing.

Appendix B.2 Uniqueness of the equilibrium in the first stage

We define the domain of the prediction vector attached with the intention: Δ^{3K} which is the simplex over the entry patterns for each its own entry result. The full prediction vector, P, is the stack of the prediction vectors attached with every intention and so its domain is $\mathcal{D} \equiv \Delta^{3K} \times \Delta^{3K} \times \Delta^{3K}$. $\mathcal{D}$ is a compact set in $\mathbb{R}^{9K}$. We define a function $\Psi : \mathcal{D} \to \mathcal{D}$ as follows:

$$\Psi(P) \equiv RQ(m(E(P))).$$

$E(P)$ is a three dimensional vector containing the expected ex ante payoffs obtained in the second stage auction: i.e., $E(P) = \left(v_{JV}^{P^{JV}}, v_S^{P^S}, v_N(P^N)\right)$. We redefine $m(E(P))$ is the two dimensional vector which includes the probability of choosing JV and S as intention. We omit the third elements because that should be on a simplex.

What we show is Ψ is a contraction mapping. Then, by Banach fixed point theorem, we know that the prediction vector satisfying the equilibrium condition is unique: in other words, there is a unique $P^\star$ such that $P^\star = \Psi(P^\star)$. For Ψ to be a contraction mapping, it is sufficient to argue that (1) Ψ has continuous partial derivative and (2) the matrix norm of its Jacobian is bounded above by 1. In our model, the mapping Q, m and E are all differentiable and so the first point is satisfied. So the remaining task is to show the second point.

We consider $L1$ matrix norm which is defined as follows: for a matrix $A \in \mathbb{R}^{n\times n}$,

$$\|A\|_1 \equiv \sup_{x\neq 0} \frac{\|Ax\|_1}{\|x\|_1}.$$

It is well known that $\|A\|_1$ is computes as follows:

$$\|A\|_1 = \max_{1\leq j\leq n} \sum_{i=1}^{n} |a_{ij}|.$$

Using this norm we evaluate the Jacobian of Ψ.

First, we have the following decomposition of the Jacobian of Ψ:

$$\nabla_P \Psi(P) = R \nabla_m Q(m) \nabla_E m(E) \nabla_P E(P)$$

Because we assume that Q is the probability vector generated by a multinomial distribution, $\nabla_m Q(m)$ is computed as follows:

$$\nabla_m Q(m(P)) = n\left(\tilde{Q}^{JV} - \tilde{Q}^N; \tilde{Q}^S - \tilde{Q}^N\right),$$

where for every $I \in \{JV, S, N\}$, $\tilde{Q}^I$ is the distribution over the intention patterns when the total number of potential entrants is less than the current situation by one. Note that its support is still set to all the intention patterns of the original number of potential entrants. These three distributions are different only in the location of non zero entries. Second, based on the Logit choice probabilities, we have the following:

$$\nabla_E m(E(P)) = \begin{pmatrix} m(JV)(1-m(JV)) & -m(JV)m(S) & -m(JV)m(N) \\ -m(S)m(JV) & -m(S)(1-m(S)) & m(S)m(N) \end{pmatrix}. \tag{11}$$

Lastly,

$$\nabla_P E(P) = U$$

where the first row of U contains all the expected payoffs in the second stage auction for every entry patterns when the firm chooses JV as its intention, and the second and third rows are set in a similar manner.

Then, we evaluate the norm of the Jacobian. We define the maximum element in U by $\bar{u}$.

$$\begin{aligned}
\|\nabla_P \Psi(P)\|_1 &= \|R \nabla_m Q(m) \nabla_E m(E) \nabla_P E(P)\|_1 \\
&\leq \|R\|_1 \|\nabla_m Q(m)\|_1 \|\nabla_E m(E)\|_1 \|\nabla_P E(P)\|_1 \\
&= 1 \times n \|\tilde{Q}^{JV} - \tilde{Q}^N; \tilde{Q}^S - \tilde{Q}^N\|_1 \times \|\nabla_E m(E)\|_1 \times \|U\|_1 \\
&< 1 \times 2n \times \frac{1}{2} \times \bar{u} \\
&= n\bar{u}.
\end{aligned}$$

For the fourth line we first bound $\|\tilde{Q}^{JV} - \tilde{Q}^{N}; \tilde{Q}^{S} - \tilde{Q}^{N}\|_1$. Because $\tilde{Q}^{JV}$, $\tilde{Q}^{S}$ and $\tilde{Q}^{N}$ are probability distribution and the sum of its elements is equal to 1, from the triangle inequality, we have $\|\tilde{Q}^{JV} - \tilde{Q}^{N}; \tilde{Q}^{S} - \tilde{Q}^{N}\|_1 < 2$. Then, we compute the upper bound of $\|\nabla_E m(E)\|_1$. From (11), the upper bound of the matrix norm is obtained as

$$\max\{m(JV)(1 - m(JV) + m(S)),\ m(S)(1 - m(S) + m(JV)),\ m(N)(1 - m(N))\}.$$

Under the constraint that $m(JV) + m(S) + m(N) = 1$ and $0 < m(JV), m(S), m(N) < 1$, by solving the constrained maximization problem, we can show that

$$m(JV)(1 - m(JV) + m(S)) < \frac{1}{2},\ m(S)(1 - m(S) + m(JV)) < \frac{1}{2},\ m(N)(1 - m(N)) < \frac{1}{4}.$$

Then we know that

$$\|\nabla_E m(E)\|_1 < \frac{1}{2}.$$

Hence, by normalizing the scale of the expected payoffs in the second stage auction so that $n\bar{u} < 1$, Ψ is a contraction mapping on $\mathcal{D}$ and by Banach fixed point theorem the equilibrium of our system uniquely exists.