Evaluating the Efficiency of Cap-Based Regulation in Matching Markets with Distributional Disparities *

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Abstract

Cap-based regulations are a commonly employed policy tool to address distributional disparities in matching markets. This paper develops a theoretical and empirical framework to evaluate the effectiveness of such cap-based regulations by integrating regional constraints into a transferable utility matching model. Using novel data from the Japan Residency Matching Program, we estimate participants' preferences and simulate counterfactual matching outcomes under various policy interventions. Simulation results reveal that cap-based regulations lead to significant efficiency losses, whereas a small subsidy achieves distributional targets still with welfare gain.

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1 Introduction

In real-world matching markets, matching outcomes often diverge from socially desirable results due to externalities, creating a gap between ideal and equilibrium matchings. To address these distributional imbalances, policymakers may implement caps and quotas as intervention strategies. Examples include race-based affirmative action in the United States to promote racial diversity (Ellison and Pathak, 2021), gender quotas in electoral systems worldwide to improve female representation (Besley et al., 2017) and residency matching markets with caps on urban placements (Kamada and Kojima, 2015).

In certain matching markets, policymakers can directly restrict the number of realized matches in different groups through centralized matching mechanisms. A prominent example, and the focus of this paper, is the medical residency matching, where variants of the Deferred Acceptance (DA) algorithm are employed to address distributional imbalances. However, current practices exhibit two significant limitations. First, they do not directly set floors for underserved groups; instead, they impose caps on popular groups to encourage inflow into less favored ones, potentially leading to inefficiencies.[1] Second, these methods neglect endogenous transfers within matched pairs, such as salaries, benefits, and training opportunities in residency markets, which are sensitive to policy changes. Addressing distributional imbalances solely by adjusting the DA algorithm, without considering the equilibrium adjustments in transfers, risks misrepresenting actual market behaviors.

This paper develops a framework for optimal taxation policy in transferable utility matching markets under regional constraints, encompassing both caps and floors. Under the optimal taxation policy, the policymaker achieves the highest social welfare among the stable outcomes that adhere to the regional constraints. Furthermore, we extend the framework to the setting introduced by Galichon and Salanié (2021*a*), which enables the identification of model primitives based solely on the aggregate-level data and simulations of equilibrium matching outcomes under a set of counterfactual scenarios. We apply this framework to Japan's residency matching market using newly collected data. Our simulation highlights the inefficiency of cap-based regulations: meeting regional constraints requires significantly reducing the number of positions in urban areas, resulting in sub-

[1] This use of caps partly arises from technical constraints associated with DA-based approaches. A feasible matching that satisfies the floors may not exist as long as submitting unacceptable partners is allowed.

stantial efficiency losses. In contrast, providing small subsidies to rural areas ensures the floor conditions while achieving the higher social welfare than the current market outcome.

Japan's residency matching market serves as an ideal case for applying our model, as distributional concerns have been central to the Japan Residency Matching Program (JRMP) since its inception. Geographic disparities in physician distribution have long challenged equitable healthcare access. Various solutions have been proposed to address these imbalances, including the JRMP's implementation of a DA algorithm with restricted urban placements to incentivize matches in underserved regions. Despite progressively stricter caps on urban placements over the past decade, distributional imbalances persist, prompting the exploration of alternative strategies such as flexible DA (Kamada and Kojima, 2015).

Our dataset encompasses the number of matches between each hospital and medical school in the JRMP from 2016 to 2019, along with detailed characteristics of the hospitals and schools, including the monthly salaries paid by hospitals to the matched medical residents. Unlike the U.S. context, where factors other than salary predominantly drive preferences, our descriptive analysis indicates that salary differences significantly influence residency choices in Japan, particularly for rural placements. This observation motivates endogenizing transfer between a pair in our model.

This study models matching markets with caps and floors, referred to as *regional constraints*, by extending the matching with transferable utility model introduced by Shapley and Shubik (1971). A regional constraint is an exogenously imposed policy goal that specifies the lower and upper bounds on the number of matches in each region, which may not be satisfied in equilibrium without intervention. We introduce a policymaker who designs a taxation policy to influence equilibrium matching outcomes. Given the taxation policy, the agents form a stable outcome. The policymaker aims to design the taxation policy under which the equilibrium matching respects the regional constraints. For the sake of empirical analysis, we extend our framework to the setting introduced by Galichon and Salanié (2021*a*). This extension enables the identification of model primitives and counterfactual simulations under different taxation policies (Proposition 1) and the design of a welfare-maximizing taxation policy (Theorem 1).

For the empirical analysis, we start by defining the transfer between matched pairs and then consider a measurement model for this transfer. To this end, we model the baseline utilities of both sides for each pair of agents. The transfer from the hospital to the matched resident represents the gap between this baseline utility and the realized utility level in equilibrium. By aggregating this transfer across the pairs within each hospital, we construct an aggregate-level transfer. We then introduce a measurement model for this aggregate-level transfer, which is linear in observed monthly salary. Our estimation proceeds in two steps: first, we estimate the aggregate-level surplus split for each pair of types, following Galichon and Salanié (2021*a*); second, based on these first-step estimates, we recover the parameters in the baseline utilities and the measurement model.

The estimation results partly align with those in the existing literature while also presenting a departure from its assumptions. On the doctor's side, our estimates indicate that factors such as the distance between the hospital and the doctor's alma mater, as well as the hospital size, are significant. Furthermore, we find that the number of previous matches is an important determinant of doctors' preferences. These findings are consistent with the existing literature, which suggests that hospitals are horizontally differentiated from doctors' perspectives. On the hospital's side, we observe that hospitals exhibit horizontal preferences similar to those of doctors: doctors from distant regions are less preferred, in addition to considerations of quality. Such horizontal preference structure on the hospital side is not allowed in the existing literature, which hinders the identification of preferences from matching data.

Based on the estimates, we simulate matching markets under various settings. The first set of simulations investigates the sources of inefficiencies inherent in the current JRMP mechanism. Removing the caps on matches in urban counties substantially enhances social welfare by increasing the number of matches in these areas. While a few rural counties fail to meet floor conditions under this scenario, these can be satisfied through small targeted subsidies which result in the higher social welfare than the current market outcome. The second set of simulations evaluates the efficiency of the flexible DA mechanism (Kamada and Kojima, 2015), a recent approach for addressing regional constraints. The results still indicate that, even with a flexible DA, aiming to resolve distributional imbalances through cap-based regulation entails too many sacrifices compared to monetary interventions. Achieving the floor conditions in rural counties requires a roughly 34% reduction in capacity in urban counties. This leads to a welfare loss compared to the current market outcome.

Related literature Matching with constraints, initiated by Kamada and Kojima (2015), has received considerable attention in various fields due to its applicability in real-world scenarios. In economics, in addition to the medical residency matching program (Kamada and Kojima, 2015), we can find theoretical analysis of school choice problem under variants of affirmative actions (Abdulkadiroğlu and Sönmez, 2003; Ehlers et al., 2014; Kojima, 2012; Hafalir, Yenmez and Yildirim, 2013). Most studies on constrained matching markets adopt a non-transferable utility model, where preferences over potential partners are exogenously defined outside the centralized mechanism. However, in many markets, transfers—such as wages—are endogenously determined in equilibrium, influenced by the mechanism itself, thereby impacting the match values between different pairs. As mentioned in the Introduction, this endogeneity concern is particularly severe in our markets. This necessitates an endogenous treatment of transfers, prompting us to consider a transferable utility matching model under constraints.

Transferable utility matching model also has long tradition of studies. As classical results, Shapley and Shubik (1971) provided the definition of stable outcome as an equilibrium concept and its characterization from the view of the linear programming, and Becker (1973) analyzed the sorting patterns under the equilibrium. Building on these results, the transferable utility matching model has been widely accepted as the foundational model describing partner search between two groups, particularly in labor economics. This framework allows for analyzing the resulting sorting and matching patterns. For example, they have been used in analyses of labor markets, such as firms and workers or firms and CEOs (Gabaix and Landier, 2008; Eeckhout, 2018), as well as in family economics in the context of marriage markets (Chiappori, 2017). Constraints like regional restrictions have not traditionally been considered in the transferable utility matching models, largely because the applications focused on markets where distributional concerns were absent. Incorporating constraints in the model is not unnatural when such concerns are present, and addressing this point is a key contribution of this study.

Next, we highlight our contribution from the viewpoint of the structural estimation of matching marekts. The structural analysis of matching markets is widely accepted in many fields of economics: in addition to the above-mentioned labor economics, industrial organization also adopts a type of matching model to describe the trade network (Fox, 2018; Fox, Yang and Hsu, 2018). Notable methodological contributions are developed in Galichon and Salanié (2021*a*): the nonparametric identification result of the social surplus function in a transferable utility matching market, which is robust to distributional assumptions on unobserved heterogeneity, along with the corresponding estimators[2]. This study builds on the general framework of Galichon and Salanié (2021*a*), proposing an extended model that accommodates regional constraints. Our framework is thus applicable without assuming a specific distributional form for unobserved heterogeneity terms. Furthermore, diverging from Galichon and Salanié (2021*a*), we propose a formal estimation strategy that exploits a measurement of the transfer to quantify the parameter values in a monetary unit.

Lastly, we mention Agarwal (2015) as the closest research to this study. Agarwal (2015) takes a different approach to analyzing the doctor-hospital matching market in the U.S. National Residency Matching Program (NRMP), where a centralized mechanism determines the matchings and salaries are determined almost exogenously. While Agarwal (2015) addresses the endogeneity of salaries using a control function approach, our study fully models the salary determination process. This difference reflects the variation in market environments: in Japan, the concentration in urban areas presents more severe issues, and salaries are used as a tool to attract more candidates. Notably, our approach allows for a more flexible preference structure than the vertical preferences assumed on the hospital side to identify the model of Agarwal (2015). Our empirical results support the presence of horizontal heterogeneities in the preferences of the hospital side.

Layout The rest of this paper is organized as follows. Section 1.1 provides the institutional background of the JRMP. Section 2 details the data sources, key variables, and descriptive statistics. Section 3 introduces the theoretical model, extending the classic matching with a transferable utility framework to incorporate regional caps and floors as policy constraints. Section 4 presents the main theoretical results, including the design of optimal taxation policies under regional constraints. Section 5 outlines the empirical strategy, describing the estimation of structural parameters and the construction of moment conditions. Section 6 provides the results of our estimation. Section 7 conducts counterfactual analyses to evaluate the efficiency of the current policy and the potential impact of alternative policies.

[2]Galichon and Salanié (2021*a*) is also distinct compared to other methodologies dependent on the distributional assumptions, such as the minimum score estimator proposed by Fox (2010, 2017); Fox and Bajari (2013) and the maximum likelihood estimator of Choo and Siow (2006).

1.1 Institutional Background of the JRMP

Established in 2004, the Japan Residency Matching Program (JRMP) is modeled after the National Resident Matching Program in the United States. The JRMP uses a deferred acceptance algorithm to match medical students seeking clinical training with hospitals offering residency programs based on mutual preferences. Typically, sixth-year medical students who plan to take the national exam participate in the program.[3]

The JRMP process is structured as follows: Students and hospitals must register for the system and submit their preferences. Students have access to information such as hospital size, location, training program details, salary, and workload, and they can participate in job fairs and visit hospitals for more details. Before submitting their preference lists, students must take exams conducted by each hospital they wish to list. After the initial submission of preference lists, the distribution of students' first-choice hospitals is disclosed once, allowing both students and hospitals to adjust their preferences before finalization. The deferred acceptance algorithm then runs to determine the matches based on the finalized preference lists.[4] Unmatched students can reach out to hospitals with vacant slots individually or wait to participate in the matching process the following year. In 2023, the JRMP saw the participation of 10,202 students and 1,209 hospitals offering 10,895 positions. The algorithm successfully matched 87.9% of the students, with 64.3% securing their first-choice hospitals, 16.3% their second choice, and 9.0% their third choice.

Although internships were not mandatory before 2004, most medical students still undertook them, typically at hospitals affiliated with their medical schools. For example, in 2001, 71.2% of students worked in such affiliated hospitals. These hospitals often offered poor financial compensation, forcing many students to take part-time jobs to cover their living expenses. Since 2004, two-year internships have become mandatory for becoming clinicians. The government began subsidizing hospitals offering residency programs, ensuring that residents receive sufficient salaries and eliminating the need for part-time work.[5] Consequently, an increasing number of students are now choosing to

[3]In Japan, students can enter a six-year medical program immediately after graduating from high school, so sixth-year students, who are in their final year, are typically 24 to 25 years old.

[4]At this stage, it is nearly impossible for students or hospitals to add more options to their preference lists. Given the (one-sided) strategy-proofness of the DA algorithm, the usefulness of this information disclosure is questionable.

[5]Another primary goal of the new internship system is to introduce a rotating internship system, allowing residents to gain a broader understanding of primary care. Before 2004, residents typically

work in non-university hospitals, with 69.27% doing so between 2017 and 2019.[6](Kitamura and Takagi, 2006)

Many argue that the introduction of the JRMP has contributed to significant geographic imbalances in the distribution of doctors. A widely accepted opinion is that when most doctors were affiliated with universities, the universities had strong control over doctor placements and could send some of them to underserved areas. However, under the JRMP, more doctors complete their internships in urban and non-university hospitals, loosening the connections between doctors and universities and preventing universities from functioning as an adjustment mechanism.[7] To compensate for the lack of residents as a labor force, university hospitals often need to request doctors working at other related non-university hospitals to return, further reducing the number of doctors in underserved areas.[8] Additionally, hospitals participating in the JRMP are required to have enough doctors to supervise residents, leading to a concentration of doctors in certain hospitals. (Endo, 2019)

To address distributional imbalances, the JRMP began implementing regional caps in 2010.[9] The regional caps are set through a systematic process: first, the total number of positions in the country is determined by multiplying the total number of medical students by a constant. This constant was approximately 1.22 in 2015, but the government plans to reduce it to 1.05 by 2025. Once the total number of positions is determined, they are allocated to each prefecture based on variables such as population, medical school capacity, current number of doctors, and geographic factors like the number of doctors per

concentrated on their specific areas of interest without receiving comprehensive training in other fields.

[6]The popularity of non-university hospitals may be attributed to better working environment, greater opportunities for practical experience with common diseases, and increased flexibility in location.

[7]Due to the competitive nature of medical schools, many students from urban areas opt to attend medical schools in rural regions. These students are more likely to choose to work in hospitals near their home areas.

[8]University hospitals want to have many medical staff members for both clinical and research purposes, but financial constraints make this difficult. Therefore, they send staff to non-university hospitals on temporary assignments, with the non-university hospitals paying their salaries. Non-university hospitals accept this arrangement because it provides a stable supply of doctors, or they request such arrangements due to doctor shortages. Although interns lack sufficient knowledge and skills to be effective in underserved areas, they can handle some tasks at university hospitals, reducing the workload of mid-level doctors and enabling these doctors to be dispatched elsewhere. Consequently, a decrease in the number of residents restricts the dispatch of mid-level doctors.

[9]Another possible approach could be to simply increase the number of doctors. However, the Japanese government opted not to pursue this strategy and instead set caps on the total number of medical students nationwide. This decision was based on the expectation that Japan's population will decrease in the coming years, potentially leading to an oversupply of doctors. An oversupply could result in physician-induced demand and increased public expenditure on health insurance.

unit area and the population of isolated islands. The allocation rule is designed to favor underserved areas, with positions in urban areas being reduced more significantly. The rationale behind these caps is that by tightening the total capacity and limiting positions in urban areas, more students will secure placements in rural regions and remain there after their internships.

2 Data

Our analysis covers the four years of matching results generated by the JRMP from 2016 to 2019. To estimate our model, we need three key elements: the matching patterns between medical schools and hospitals, the characteristics of these institutions relevant to the preferences of medical students and hospitals, and the salaries paid to residents during their internships. We begin by explaining the data sources and then present the descriptive statistics in Section 2.1.

The matching patterns between medical schools and hospitals (i.e., the number of matches between any given pair) are derived from the "Physician Registration Report," which includes information such as doctors' registration numbers, workplace postal codes, and the universities from which they graduated. This data allows us to determine the annual number of matches between specific medical schools and hospitals.

We obtained the characteristics of hospitals from the JRMP website, which provides details such as hospital names, program offerings, and capacity. For the characteristics of medical schools, we used the national exam pass rate and whether the university is public, based on publicly available information from hospital websites. Additionally, to measure the expected ability of graduates from a medical school, we used the T score of the entrance exam.[10] T scores are widely recognized as an indicator of university entrance exam difficulty in Japan, with higher scores indicating more challenging universities. We used the most recent T scores available for our estimation.[11]

Finally, we gathered salary data by crawling hospital websites. Due to limited data availability, we used the most recent salary information rather than data from 2016 to 2019, assuming that salary levels remained constant during this period. We also collected

[10]The T scores of universities are published by cram schools. These scores are calculated using data from practice exams administered by the cram schools, which gather information on students' actual university entrance exam results. The T scores reflect the relationship between students' practice exam performance and their success in university entrance exams.

[11]The data source is https://www.keinet.ne.jp/university/ranking/.

Table 1. Environments and Outcomes of JRMP

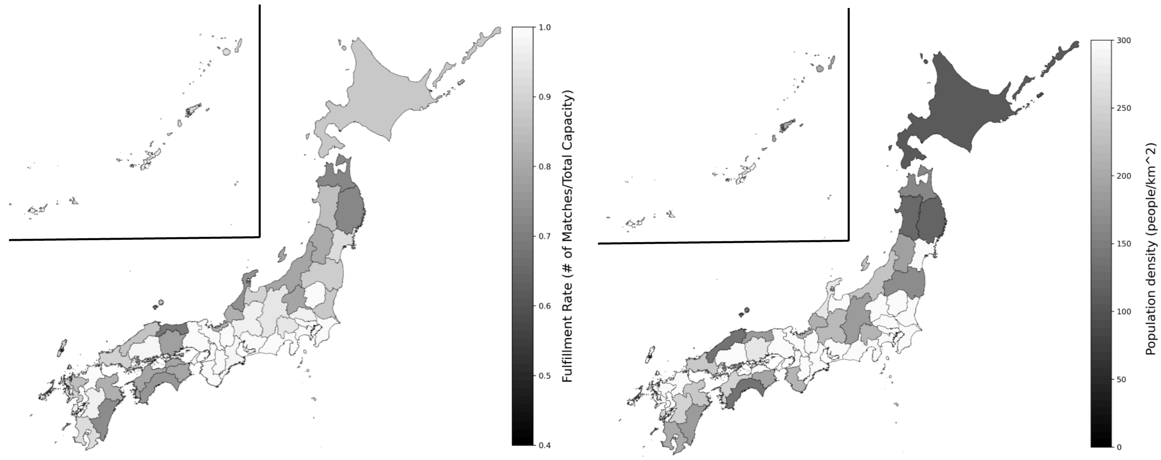
	2017	**2018**	**2019**
Panel A: Doctor side			
Number of schools	78	78	78
Number of students	9830	9916	9932
Number of matched students	8530	8369	8634
Number of unmatched students	1300	1547	1298
Unmatch rate (%)	13.22	18.48	15.03
Panel B: Hospital side			
Number of hospitals	1025	1025	1022
Number of total seats	11716	11468	11730
Number of matched seats	8530	8369	8634
Number of unmatched seats	3186	3099	3096
Unmatch rate (%)	27.19	27.02	26.39
Number of excess seats	1886	1551	1798
Excess rate (%)	16.01	13.52	15.33
Panel C: Monthly salary ($1,000$ JPY)			
Avg.	386.4	386.4	386.1
Std.	99.5	99.6	99.4
Min	180.0	180.0	180.0
Max	855.0	855.0	855.0

additional hospital-related information, such as location, number of beds, and emergency transport cases.

2.1 Descriptive Statistics

Table 1 summarizes the environment and the outcomes of JRMP for the years 2017, 2018, and 2019. Since our estimation uses the matching patterns from the final year as one of the covariates, we focus on these three years. Panel A and Panel B in Table 1 show the results of JRMP. While the environment, including the total number of medical students and the available seats, and their ratios, exhibits minor yearly variations, the matching outcomes—such as the number of matches, unmatches, and the unmatch rate—fluctuate over this period. As noted in Section 1.1, the unmatch rate had been increasing before this period due to the tightening caps in urban areas. Although the unmatch rate remains high, our data cover a relatively stationary market.

Despite the presence of unoccupied seats overall, as shown in Table 1, the fulfillment rate by prefecture, defined as the ratio of the number of matches to the total number of



(a) Fulfillment Rates (b) Population Density

Figure 1. Fulfillment Rate and Population Density

positions in each prefecture, exhibits substantial regional variation. Figure 1a displays a choropleth map of fulfillment rates, while Figure 1b illustrates population densities by prefecture in 2019. As noted in Section 1.1, these figures suggest that rural regions, characterized by lower population density, are less popular and suffer lower fulfillment rates than urban areas.

Panel C of Table 1 reports significant variation in the salaries of medical interns across Japan. For example, in 2017, the average annual salary was approximately \$31,714, with a standard deviation of \$8,166.[12] For comparison, Table 1 of Agarwal (2015) reports that the mean salary for similar medical interns in the United States was \$47,331, with a standard deviation of \$2,953. While average salaries are higher in the U.S., the standard deviation in Japan is 2.77 times larger, indicating greater salary dispersion.

Table 2 presents descriptive statistics for the covariates used to parameterize social surplus and the utility of medical students and hospitals. The T score and graduation exam pass rate of medical schools serve as proxies for student ability, with higher values being more desirable to hospitals. Public medical schools exhibit higher average T scores, indicating that graduation from a public school signals greater ability. This difference is statistically significant. On the hospital side, we define an indicator for urban hospitals located in the six prefectures with officially set caps on the number of matches in the JRMP: Tokyo, Kanagawa, Aichi, Kyoto, Osaka, and Fukuoka. Hospital size is proxied by the number of beds, with Table 2 showing that university hospitals and those in urban

[12] The conversion from yen to dollars was based on the exchange rate as of August 30, 2024.

Table 2. Summary Statistics of Covariates

	Count	Mean	Std	Min	Max
T score					
Private University	27	64.96	2.17	62	72
Public University	51	66.38	2.66	63	74
Exam pass rate					
Private University	27	0.93	0.05	0.79	1.00
Public University	51	0.93	0.03	0.82	1.00
Number of Beds					
Non University hospital	911	411.08	147.73	36	1097
University hospital	121	626.74	272.77	295	1379
Rural Hospital	684	417.99	155.92	36	1195
Urban hospital	348	472.48	217.84	38	1379

areas tend to be larger.

Public medical schools in Japan play a pivotal role in maintaining standardized medical services nationwide, with a significant proportion of their graduates continuing to nearby hospitals for internships. Figure 2 illustrates the matching pattern between hospitals and medical schools in 2017, where each cell represents a pairing, with black cells indicating at least one match. Hospitals are indexed according to the official prefectural index set by the Japanese government, ordered by location, and within each prefecture, by latitude. Medical schools are first divided into public and private groups, with the same ordering method applied within each group.

The upper half of Figure 2 (up to index 50) represents the matchings of public schools, while the lower half corresponds to private schools. Both public and private schools generally tend to match with nearby hospitals; however, this tendency is more pronounced for public schools. Consequently, public school graduates are, on average, less likely to match with urban hospitals despite the higher average quality of public universities.

2.2 Endogeneity of Salary

The matching patterns observed above cannot be attributed solely to the geographic preferences of hospitals and schools; another plausible explanation is that rural hospitals offer more attractive conditions to draw students from public universities. In this section,

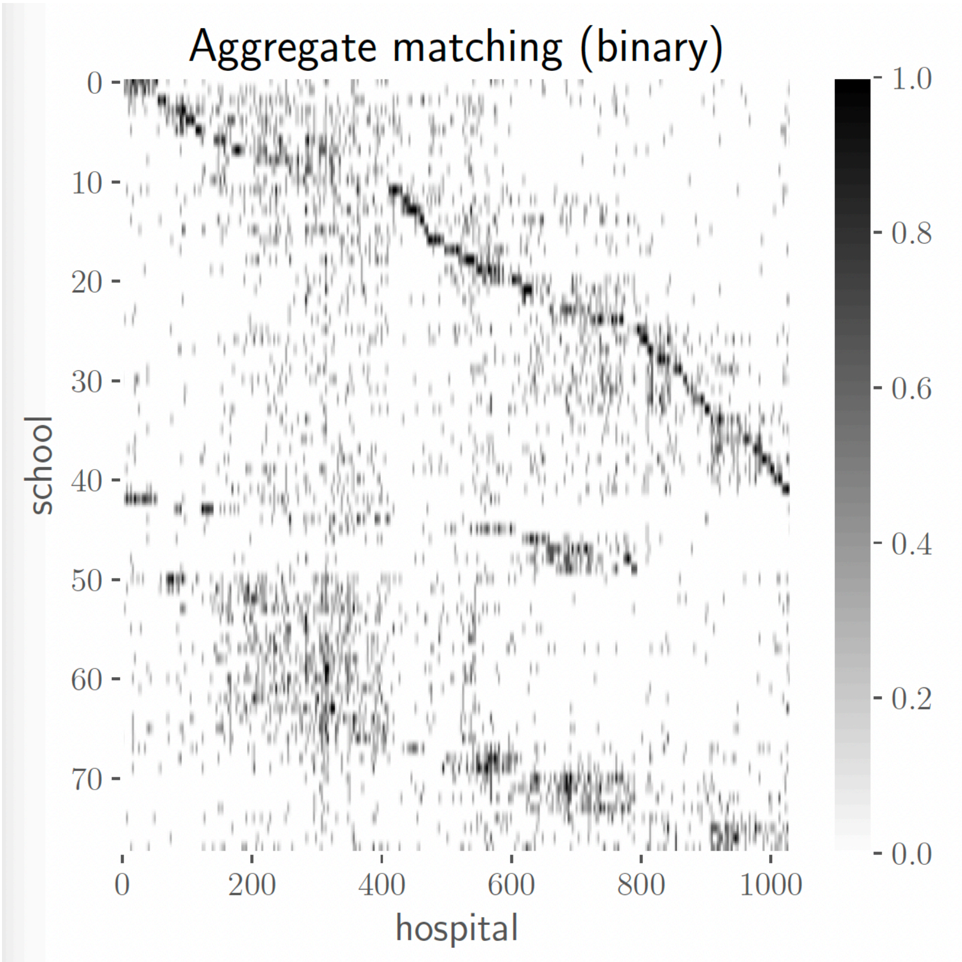


Figure 2. Binarized Matching Patterns between All Schools and Hospitals in 2017

Table 3. Sorting between Residents and Programs from Program Viewpoint

	(1) Public	(2) T score	(3) Pass rate
ln(Beds)	0.0665***	7.060***	0.0990***
	(0.0225)	(1.242)	(0.0171)
ln(Wage)	0.00233	-3.459*	-0.0376
	(0.0364)	(1.974)	(0.0276)
University hospital	-0.282***	-1.704	-0.0150
	(0.0343)	(1.116)	(0.0155)
Urban	-0.110***	3.399***	0.0435***
	(0.0178)	(0.863)	(0.0120)
N	3096	3096	3096
FE	√	√	√

Standard errors in parentheses

* p<0.1, ** p<0.05, *** p<0.01

we examine how salary functions as a compensating differential to attract more applicants to rural areas and explore evidence suggesting that salary should be treated as an endogenous variable in our model.

We conduct a series of regressions to investigate (i) which types of hospitals attract better students and (ii) which types of students are matched with better hospitals, similar to Agarwal (2015). Due to the unavailability of individual matching data, we first calculate the weighted averages of the characteristics of matched partners within each medical school and hospital. These aggregated characteristics are then regressed on the covariates of the medical schools and hospitals.

Table 3 presents the regression results on sorting from the perspective of hospitals. Column (1) shows that urban hospitals tend to have more students from private schools, which are generally of lower quality compared to public schools. Meanwhile, columns (2) and (3) indicate that hospitals in urban areas and those with a larger number of beds are more likely to attract students from higher-quality schools, as measured by T scores and pass rates. Table 4 presents the regression results from the perspective of medical schools. Columns (1), (2), and (4) indicate that students from public schools are more likely to be matched with smaller hospitals in rural areas, where salaries tend to be higher

Table 4. Sorting between Residents and Programs from Residents Viewpoint

	(1) Beds	(2) Univ. Hospital	(3) Urban Hospital	(4) ln Wage
Public University	-153.1***	-0.309***	-0.302***	0.136***
	(17.60)	(0.0334)	(0.0438)	(0.0250)
T score	2.935	-0.0145**	0.0427***	-0.00932**
	(3.056)	(0.00685)	(0.00888)	(0.00377)
Exam Pass Rate	-47.03	0.0945	0.0256	0.0563
	(58.28)	(0.149)	(0.114)	(0.0724)
N	234	234	234	234
FE	√	√	√	√

Standard errors in parentheses
* p<0.1, ** p<0.05, *** p<0.01

than in urban hospitals.

The matching patterns do not fully align with positive assortative matching, as high-quality public schools play a unique role in providing medical residents to local hospitals. These observations support the hypothesis that rural hospitals offer higher salaries to attract more applicants, particularly those from nearby public universities. Therefore, when simulating outcomes of this matching market as part of the counterfactual analysis, it is essential to account for how transfers between hospitals and matched residents are determined in the counterfactual equilibrium. This consideration is a reason why we do not adopt a non-transferable utility matching model for the following analysis, as it treats transfers as exogenous elements.

3 Model

Stable outcome under a taxation policy We consider a two-sided matching market. Let I denote the set of doctors (medical students) and J the set of job slots owned by hospitals.[13] Each doctor $i \in I$ can be matched with at most one slot $j \in J$, and each slot can accommodate at most one doctor. If a doctor i is unmatched, they are paired with

[13]Here, $j \in J$ refers to a job slot within a hospital. Each hospital can have multiple job slots, and aims to maximize the aggregate payoffs from these slots, implying that hospitals' preferences are *responsive* (Roth and Sotomayor, 1990).

an outside option j_0. Similarly, an unmatched slot j is paired with an outside option i_0. A *matching* is represented by a 0-1 matrix $d = (d_{ij})_{i\in I, j\in J}$, where $d_{ij} = 1$ if and only if doctor i is matched with slot j. A matching d is *feasible* if each doctor is matched to exactly one slot or the outside option, and each slot is matched to exactly one doctor or the outside option: $\sum_{j\in J} d_{ij} \leq 1$ for all $i \in I$, and $\sum_{i\in I} d_{ij} \leq 1$ for all $j \in J$.

There is a policymaker who faces an additional condition called regional constraints. There are L regions, denoted by $Z = \{z_1, z_2, \ldots, z_L\}$, with each job slot j assigned to one region. Additionally, we define a special region z_0 that contains only the outside option j_0. With a slight abuse of notation, let $z\colon J \cup \{j_0\} \to Z \cup \{z_0\}$ be the mapping where $z(j)$ indicates the unique region to which job slot j belongs. Each region z has a cap and a floor, $\underline{o}_z \in \mathbb{R}_+$ and $\bar{o}_z \in \mathbb{R}_+ \cup \{\infty\}$, respectively. We say a feasible matching d satisfies *regional constraints* if it respects the caps and the floors: $\sum_{i\in I}\sum_{j\in z} d_{ij} \in [\underline{o}_z, \bar{o}_z]$ for each $z \in Z$. Throughout the paper, we assume that there exists at least one feasible matching that satisfies regional constraints.

Agents form a stable outcome à la Shapley and Shubik (1971). Without policy intervention, the realized matching may not meet the regional constraints. The policymaker can implement a taxation policy that influences the split of the joint surplus among agents to satisfy the regional constraints. When a doctor i and a slot j are matched, they generate an *(individual-level) net joint surplus* $\Phi_{ij} \in \mathbb{R}$. The tax $w_z \in \mathbb{R}$ is imposed on each match (i, j) in region z with negative taxes being interpreted as subsidies. We assume $w_{z_0} = 0$, i.e., no tax is imposed on the outside option. With taxation policy w, each matched pair divides the *gross joint surplus* $\Phi_{ij} - w_{z(j)}$ instead of the net joint surplus. [14] The stable outcome under a taxation policy is defined as follows:

Definition 1 (Stable outcome). *Given the matching market (I, J, Z, z, Φ),*[15] *a profile $(d, (u, v))$ of feasible matching d and equilibrium payoffs (u, v) forms a* stable outcome *under taxation policy w if it satisfies:*

1. *Individual rationality: For all $i \in I$, $u_i \geq \Phi_{i,j_0}$, with equality if i is unmatched. For all $j \in J, v_j \geq \Phi_{i_0,j}$, with equality if j is unmatched.*

[14]In principal, the policymaker may want to impose different amounts of taxes on distinct pairs within the same region. Although we exclude such possibilities in the definition of taxation policy, we can show that such a restriction is harmless in terms of social welfare: there is a welfare-maximizing taxation policy that imposes the same amount of tax on all the pairs in the same region.

[15]The symbol z denotes the mapping from job slots to regions.

2. No blocking pairs: For all $i \in I$ and $j \in J$, $u_i + v_j \geq \Phi_{ij} - w_{z(j)}$, with equality if $d_{ij} = 1$.

We say d is a stable matching *if there exists (u, v) such that $(d, (u, v))$ forms a stable outcome.*

Unobserved heterogeneity Let $X = \{x_1, x_2, \ldots, x_N\}$ represent the finite set of observable characteristics, or *types*, of doctors. Each doctor $i \in I$ has a type $x(i) \in X$. Similarly, let $Y = \{y_1, y_2, \ldots, y_M\}$ represent the finite set of observable characteristics of job slots, with each slot $j \in J$ having a type $y(j) \in Y$. Although agents with the same type are indistinguishable to the policymaker, there can be *unobservable heterogeneity*: doctors of the same type x or job slots of the same type y may generate different joint surpluses when matched. For convenience, we denote $i \in x$ if $x(i) = x$ and $j \in y$ if $y(j) = y$. We define x_0 and y_0 as special types representing the outside options i_0 and j_0, respectively, and let $X_0 = X \cup \{x_0\}$ and $Y_0 = Y \cup \{y_0\}$ include these outside options. The set of all type pairs is denoted by $T = X_0 \times Y_0 \setminus \{(x_0, y_0)\}$. We assume each job slot type $y \in Y$ belongs to a unique region, denoted by $z(y) \in Z$. Let n_x be the number of doctors with type x, and m_y be the number of job slots with type y.

Let μ_{xy} denote the number of matches between type-x doctors and type-y job slots, defined as $\mu_{xy} = \sum_{i \in x} \sum_{j \in y} d_{ij}$. An *aggregate-level matching* $\mu = (\mu_{xy})_{x \in X, y \in Y}$ is said to be *feasible* if it satisfies the population constraints $\sum_y \mu_{xy} = n_x$ and $\sum_x \mu_{xy} = m_y$ for each x and y. Furthermore, we say μ satisfies regional constraints if $\sum_{y \in z} \sum_{x \in X} \mu_{xy} \in [\underline{o}_z, \bar{o}_z]$ for each $z \in Z$.

Remark 1. *(Interpretation of joint surpluses) The joint surplus includes not only the revenue generated by the match but also other potential gains such as experience and knowledge. Thus, Φ_{ij} represents the value of all such potential gains, measured in a numeraire.*

Remark 2. *(Validity of stable outcome) It is reasonable to assume that participants form a stable outcome in various matching markets such as in frictionless decentralized matching markets (e.g., certain labor markets or marriage markets.[16]) Additionally, we can show that agents may form a stable outcome in a game where (i) hospitals first set wages, (ii) agents submit their preference lists after observing wages, and (iii) the matching is*

[16] See, for example, Roth and Sotomayor (1990) and Chiappori (2017) for textbook references.

determined by the deferred acceptance algorithm, which is a good approximation of the JRMP (see Appendix A.5 for more details.)

Remark 3. *(Interpretation of types and regions) Types $y \in Y$ and regions $z \in Z$ can be interpreted in various ways. For example, in our application, a type y may correspond to a hospital, and a region z may correspond to a district (e.g., a prefecture). In other contexts, a type could represent a subcategory of occupation (e.g., registered nurse, physician assistant), and a region could represent a broader occupational category (e.g., healthcare).*

4 Theoretical Results

Discrete choice representation We here review a set of conditions and results, developed by Galichon and Salanié (2021*a*), that connect the individual-level objects $((\Phi_{ij})_{ij}, (d_{ij})_{ij}, (u_i)_i, (v_j)_j)$ introduced in Section 3 to the aggregate-level objects we develop in this section.

For a pair (i, j) with $i \in x$ and $j \in y$, we assume the individual-level joint surplus Φ_{ij} can be decomposed into the sum of the *aggregate-level joint surplus* Φ_{xy} and independent mean-zero error terms ε_{iy} and η_{xj}. [17] For each x and $i \in x$, error term $(\varepsilon_{iy})_{y \in Y_0}$ is drawn from the distribution $P_x \in \Delta(\mathbb{R}^{|Y|+1})$. Similarly, for each y and $j \in y$, error term $(\eta_{xj})_{x \in X_0}$ is drawn from distribution $Q_y \in \Delta(\mathbb{R}^{|X|+1})$.

Assumption 1 (Independence)**.** *The error terms are independent across all i and j and of mean-zero.* [18]

Assumption 2 (Additive Separability)**.** *There is a matrix $(\Phi_{xy})_{(x,y) \in T}$ such that (i) $\Phi_{ij} = \Phi_{xy} + \varepsilon_{iy} + \eta_{xj}$ for each $x \in X$, $y \in Y$, $i \in x$, and $j \in y$, and (ii) $\Phi_{i,y_0} = \varepsilon_{i,y_0}$, $\Phi_{x_0,j} = \eta_{x_0,j}$ for each $x \in X$ and $y \in Y$.*

We define *aggregate-level utilities* U_{xy} and V_{xy}, which represent the equilibrium payoffs dependent solely on types, as follows: for each x and y, define

$$U_{xy} \coloneqq \min_{i \colon x(i)=x} \{u_i - \varepsilon_{iy}\}, \ V_{xy} \coloneqq \min_{j \colon y(j)=y} \{v_j - \eta_{jx}\}$$

[17] Under Assumption 1 and 2, Φ_{xy} is the average joint surplus conditional on $i \in x$ and $j \in y$.

[18] The mean-zero assumption here is without loss of generality. If not, we can always demean the error terms and redefine them.

and $U_{x,y_0} = V_{x_0,y} := 0$. The following lemma argues that the matching market can be seen as a bilateral discrete choice problem.

Lemma 1 (Galichon and Salanié (2021*a*)). *Let (u, v) be a payoff profile in a stable outcome. Under Assumption 2, for any doctor $i \in I$ and any slot $j \in J$, we have*

$$u_i = \max_{y \in Y_0} \left\{ U_{x(i),y} + \varepsilon_{iy} \right\}, \quad v_j = \max_{x \in X_0} \left\{ V_{x,y(j)} + \eta_{xj} \right\}.$$

Proof. See Appendix A.1. □

By Lemma 1, the social welfare, defined as the sum of the equilibrium payoffs, on the doctor side can be written as $\sum_{i \in I} u_i = \sum_{x \in X} n_x \cdot \frac{1}{n_x} \sum_{i \in x} \max_{y \in Y_0} \{U_{xy} + \varepsilon_{iy}\}$. When n_x is sufficiently large, the term $\frac{1}{n_x} \sum_{i \in x} \max_{y \in Y_0} \{U_{xy} + \varepsilon_{iy}\}$ can be approximated by $\mathbb{E}_{\varepsilon_i \sim P_x} [\max_{y \in Y_0} \{U_{xy} + \varepsilon_{iy}\}]$. We will assume from now on that this large market limit is a good approximation, so the social welfare on the doctor side becomes

$$G(U) := \sum_{x \in X} n_x \mathop{\mathbb{E}}_{\varepsilon_i \sim P_x} \left[\max_{y \in Y_0} \{U_{xy} + \varepsilon_{iy}\} \right].$$

Similarly, when m_y is sufficiently large for each y, the welfare on the hospital side is approximated by

$$H(V) := \sum_{y \in Y} m_y \mathop{\mathbb{E}}_{\eta_j \sim Q_y} \left[\max_{x \in X_0} \{V_{xy} + \eta_{xj}\} \right].$$

Under the assumption that the CDFs of the error terms are continuously differentiable, we have (the Williams-Daly-Zachary theorem (McFadden, 1980)):

$$\frac{\partial G}{\partial U_{xy}}(U) = \frac{\partial}{\partial U_{xy}} \mathop{\mathbb{E}}_{\varepsilon_i \sim P_x} \left[\max_{y \in Y_0} \{U_{xy} + \varepsilon_{iy}\} \right] = \Pr(i \text{ chooses } y \mid i \in x).$$

When n_x is sufficiently large, $n_x \Pr(i \text{ chooses } y \mid i \in x)$ is a good approximation of μ_{xy}.

Assumption 3 (Large Market Approximation). *For each $x \in X$, we approximate $\mathbb{E}_{\varepsilon_i \sim P_x} [\max_{y \in Y_0} \{U_{xy} + \varepsilon_{iy}\}]$ as $\frac{1}{n_x} \sum_{i \in x} \max_{y \in Y_0} \{U_{xy} + \varepsilon_{iy}\}$. We also approximate μ_{xy} as $n_x \Pr(i$ chooses type-y slot $\mid i \in x)$. Similar conditions are assumed for type $y \in Y$.*

Assumption 4 (Smooth Distribution). *For each x, y, CDF's P_x and Q_y are continuously differentiable.*

Policymaker's problem A tuple $\mathcal{M} := (X, Y, n, m, Z, z, \Phi, P, Q)$ characterizes an aggregate-level matching market, where $n := (n_x)_x$, $m := (m_y)_y$, $\Phi = (\Phi_{xy})_{xy}$, $P = (P_x)_x$, and $Q = (Q_y)_y$. Given $\mathcal{M}$, the policymaker aims to (i) compute the optimal taxation policy w that maximizes social welfare while respecting regional constraints, and (ii) compute the matching and social welfare for different taxation policies. For these goals, we will construct optimization problems that only requires knowledge of the aggregate-level surplus $(\Phi_{xy})_{x,y}$. To this end, we need an additional technical assumption on the error term distributions:

Assumption 5 (Full support)**.** $\text{supp}(P_x) = \mathbb{R}^{|Y_0|}$ *and* $\text{supp}(Q_y) = \mathbb{R}^{|X_0|}$ *for each* x *and* y*.*

This assumption guarantees that G and H are strictly convex,[19] so there is a one-to-one correspondence between an aggregate-level matching μ and aggregate-level utilities U (and similarly between μ and V.)[20] The optimization problem is now defined as follows:

$$
\text{(P)}\quad
\begin{array}{|lll}
\underset{\mu \geq 0}{\text{maximize}} & \displaystyle\sum_{(x,y)\in T} \mu_{xy}\Phi_{xy} + \mathcal{E}(\mu) & \\
\text{subject to} & \displaystyle\sum_{y\in Y_0} \mu_{xy} = n_x & \forall x \in X, \\
& \displaystyle\sum_{x\in X_0} \mu_{xy} = m_y & \forall y \in Y, \\
& \displaystyle \underline{o}_z \leq \sum_{y\in z}\sum_{x\in X} \mu_{xy} \leq \bar{o}_z \; \forall z \in Z, &
\end{array}
$$

where $\mathcal{E}(\mu) := -G^*(\mu) - H^*(\mu)$, and G^* and H^* are the Legendre-Fenchel transform of G and H, respectively.[21] We can show that (P) is a concave programming that maximizes social welfare subject to regional constraints (see Appendix A.4.) Its dual problem is

$$
\text{(D)}\quad
\begin{array}{|lll}
\underset{U,V,\bar{w}_z,\underline{w}_z}{\text{minimize}} & \displaystyle G(U) + H(V) + \sum_{z\in Z} \bar{o}_z \bar{w}_z - \sum_{z\in Z} \underline{o}_z \underline{w}_z & \\
\text{subject to} & U_{xy} + V_{xy} \geq \Phi_{xy} - \bar{w}_{z(y)} + \underline{w}_{z(y)} & \forall (x,y) \in T, \\
& \bar{w}_z \geq 0,\ \underline{w}_z \geq 0 & \forall z \in Z.
\end{array}
$$

[19] See Appendix A.3 for the proof.

[20] If G and H strictly convex, the Legendgre transform of G and H, denoted by G^* and H^*, are differentiable (Proposition D.14 of Galichon (2018).) Then, we have $\mu \in \frac{\partial G}{\partial U}(U)$ iff $U \in \frac{\partial G^*}{\partial \mu}(\mu)$ (Proposition D.13 of Galichon (2018).)

[21] G and H are proper convex functions, so their Legendre-Fenchel transforms are well-defined.

The primal problem (P) has the optimal solution since its objective function is continuous and its feasible set is compact and non-empty. Due to strong duality,[22] the dual problem (D) also has the optimal value, which coincides with that of (P). Moreover, (P) and (D) have unique solutions (see Appendix A.2.) The following theorem claims that, given regional constraints $(\bar{o}_z, \underline{o}_z)_z$, the optimal taxation policy w and a tuple of aggregate-level matching and utilities (μ, U, V) are characterized by the solutions to (P) and (D).

Theorem 1. *Assume that Assumptions 1-5 are satisfied. Fix any aggregate-level matching market $\mathcal{M}$. Suppose that μ and $(U, V, \bar{w}, \underline{w})$ are the solutions to (P) and (D), respectively.*

1. *Let $w_z := \mathbb{1}\{\bar{w}_z > 0\}\bar{w}_z - \mathbb{1}\{\underline{w}_z > 0\}\underline{w}_z$ for each z. Then, w is the optimal taxation policy, and (μ, U, V) are the corresponding aggregate-level matching and utilities.*
2. *Suppose that w is the optimal taxation policy. Then, we have $\bar{w}_z := \mathbb{1}\{w_z > 0\}w_z$ and $\underline{w}_z := -\mathbb{1}\{w_z < 0\}w_z$.*

Proof. See Appendix A.3. □

We can also compute a tuple of aggregate-level matching and utilities (μ, U, V) realized under taxation policy w for any w by solving a pair of optimization problems. The proof of the following proposition is similar to the one for Theorem 1, and so is omitted.

Proposition 1. *Assume that Assumptions 1-5 are satisfied. Fix any aggregate-level matching market $\mathcal{M}$. For any taxation policy w, the aggregate-level matching and utilities realized under w, denoted by $(\mu(w), U(w), V(w))$, is characterized by the solution to (P_w) and (D_w), defined as follows:*

$$(P_w) \quad \left| \begin{array}{lll} \underset{\mu \geq 0}{\text{maximize}} & \displaystyle\sum_{(x,y)\in T} \mu_{xy}\left(\Phi_{xy} - w_{z(y)}\right) + \mathcal{E}(\mu) & \\ \text{subject to} & \displaystyle\sum_{y\in Y_0} \mu_{xy} = n_x & \forall x \in X, \\ & \displaystyle\sum_{x\in X_0} \mu_{xy} = m_y & \forall y \in Y. \end{array} \right.$$

[22] The constraints of the primal problem satisfy the weak Slater's condition (all functions are affine in μ.)

$$(D_w) \quad \left| \begin{array}{ll} \underset{U,V}{\textit{minimize}} & G(U)+H(V) \\ \textit{subject to} & U_{xy}+V_{xy} \geq \Phi_{xy} - w_{z(y)} \quad \forall (x,y) \in T \end{array} \right.$$

For taxation policy w, tuple $(\mu(w), U(w), V(w))$ defined in Proposition 1 is called an *aggregate equilibrium (AE)* under w. It is called the *efficient aggregate equilibrium (EAE)* when the corresponding w is the optimal taxation policy.

5 Empirical Strategy

We begin by mapping the primitives and equilibrium objects in the model described in Section 3 and 4 to their empirical counterparts in the doctor-hospital matching market. We then introduce the concept of transfers between matched pairs within the matching market. To define these transfers, we impose an additional structure on the composition of the net joint surplus. Finally, we outline our empirical strategy to estimate the structural parameters. Our estimation consists of two steps: the first step, detailed in Section 5.2.1, follows Galichon and Salanié (2021a) to identify aggregate-level utilities from the observed aggregate matching; the second step, described in Section 5.2.2, utilizes a moment condition that links the observed salary to the model-induced objects to estimate the parameter values.

We use the following notation: a doctor is denoted by i, and a job slot is denoted by j. Each doctor belongs to a medical school, and each slot is offered by a hospital, with s representing a school and h representing a hospital. We consider s and h as observable types of doctors and slots, using $s(i)$ to denote the medical school to which doctor i belongs, and $h(j)$ to denote the hospital offering slot j. The matching market operates over $T \in \mathbb{N}$ periods, with t denoting each observation period. Let Z denote the set of regions and $z(h)$ denote the region to which hospital h belongs, assuming $z(h)$ remains constant over time. The aggregate-level joint surplus at time t is denoted by Φ_{sht}. The unobserved part (error term) of doctor i's preference for hospital h is denoted by ε_{iht}, while the unobserved part of slot j's preference for school s is denoted by η_{sjt}.[23] The

[23]This notation can be confusing as slots themselves do not have preferences. We interpret this as follows: the admission office is composed of members with varying tastes for schools, such as a strong preference for the school from which they graduated. η_{sjt} reflects these differences in the committee members' preferences.

net joint surplus satisfies the equality: $\Phi_{ijt} = \Phi_{s(i)h(j)t} + \varepsilon_{ih(j)t} + \eta_{s(i)jt}$ for each $i \in I$, $j \in J$, and $t \in [T]$.[24] The matching $(d_{ijt})_{i,j,t}$ is not observable. Instead, the available data comprises the aggregate-level matching $(\mu_{sht})_{s,h,t}$, which is the number of matches between medical school s and hospital h at time t.

We consider the observed number of matches in a year composes the aggregate equilibirum under no tax of the matching market of the year. In other words, we do not consider the current matching outcome is affected by any monetary intervention aimed for satisfying regional constraints.[25] Hence, except the econometric treatment of the transfer among the pair, our empirical strategy follows Galichon and Salanié (2021*a*).

5.1 Transfer

To define a model object corresponding to the observed salary, we impose an additional structure on the net joint surplus. The *base utility* of i when matched with j at time t is the utility felt by i when matching with j net of transfer, and is denote by U_{ijt}^{base}. Similarly, the base utility of j when matched with i at time t is denoted by V_{ijt}^{base}. We assume that the net joint surplus is the sum of the base utilities of a doctor and a slot forming the match.

Assumption 6. $\Phi_{ijt} = U_{ijt}^{\text{base}} + V_{ijt}^{\text{base}}$.

Furthermore, in accordance with additive separability (Assumption 2), we re-interpret the i.i.d. error terms as the unobserved taste shocks of the agents on both sides of the market. In other words, we assume the following utility structure: define $U_{sht}^{\text{base}} \coloneqq U_{ijt}^{\text{base}} - \varepsilon_{iht}$ and $V_{sht}^{\text{base}} \coloneqq V_{ijt}^{\text{base}} - \eta_{sjt}$, so we have

$$U_{ijt}^{\text{base}} = U_{sht}^{\text{base}} + \varepsilon_{iht}, \quad V_{ijt}^{\text{base}} = V_{sht}^{\text{base}} + \eta_{sjt}.$$

We call U_{sht}^{base} and V_{sht}^{base} by *aggregate-level base utility*: they are a part of base utilities which is determined by the observable characteristics. As a direct implication of Assumption 2, 6 and (5.1), we have $\Phi_{sht} = U_{sht}^{\text{base}} + V_{sht}^{\text{base}}$. Note that the aggregate-level base utility U_{sht}^{base} can be different from the aggregate-level utility U_{sht} introduced in (4).

[24] $[T] \coloneqq \{1, 2, \ldots, T-1, T\}$.

[25] In Appendix C.1, we check if this assumption is valid in our data. We estimate our model under assumption that the matching outcome composes the efficient aggregate equilibrium under a regional constraint. The estimated value of tax levied does not increase whereas the regional constraint set by the policymaker is gradually tightened. This result indicates that "implicit" taxation is not put in the current market.

We define an individual level transfer. Fix any period t. Consider a matched pair (i,j) with $h(j)=h$ and $z(h(j))=z$ for some h and z. We define *individual-level transfer from hospital h to doctor i*, denoted by τ_{iht}, as follows:

$$\tau_{iht} \coloneqq u_{it} - \left(U_{sht}^{\text{base}} + \varepsilon_{iht}\right).$$

In equilibrium, doctor i enjoys equilibrium payoff u_{it}, which could be different from U_{ijt}^{base}. We interpret the difference between equilibrium payoff and base utility as the individual-level transfer from the hospital side to the doctor side.

Now we define an *aggregate-level transfer* as the average of the individual-level transfer in a hospital h and denote it by ι_{ht}:

$$\iota_{ht} \coloneqq \frac{1}{|D(h)_t|} \sum_{i \in D(h)_t} \tau_{iht},$$

where $D(h)_t$ is the set of doctors matched with any slot of hospital h at time t, We can show the following identities: the aggregate-level transfer from a hospital is equal to the weighted average of the gap between aggregate-level utility and aggregate-level base utility. We use these identities as moment conditions to identify the aggregate-level base utility.

Proposition 2.

$$\iota_{ht} = \sum_s \omega_{sht}\left(U_{sht} - U_{sht}^{\text{base}}\right),\ \iota_{ht} = \sum_s \omega_{sht}\left(V_{sht}^{\text{base}} - V_{sht}\right)$$

where $\omega_{sht} = \frac{\mu_{sht}}{\sum_{s'} \mu_{s'ht}}$.

5.2 Estimation

Based on the observable characteristics of s and h, we have a set of variables related to the preferences: we use $X_{sht}^{U,\text{base}}$ as the variables for U_{sht}^{base}, and $X_{sht}^{V,\text{base}}$ as the variables for V_{sht}^{base}. We assume linear structure on both of the preferences: $U_{sht}^{\text{base}} = X_{sht}^{U,\text{base}\prime}\beta_U$, $V_{sht}^{\text{base}} = X_{sht}^{V,\text{base}\prime}\beta_V$. Our parameters of interest are β_U and β_V. We use θ to indicate the vector of these parameters: $\theta \coloneqq (\beta_U, \beta_V)$.

Our estimation consists of the following two steps:

1. Estimate the aggregate-level utilities U_{sht} and V_{sht} for every t, and then

2. Estimate θ using the estimated aggregate-level utilities and the observed salaries.

5.2.1 First step

Despite the nonparametric identification results obtained in Galichon and Salanié (2021*a*), we estimate the parametrized version of the aggregate-level utilities. This is because some pairs of school and hospital have zero matches in practice. Hence, in the first step, we use the moment matching estimator proposed in Galichon and Salanié (2021*a*) to estimate U_{sht} and V_{sht}.[26]

By formulating the aggregate matching outcome μ_{sht} as a realization of a Poisson distribution, it is possible to estimate the aggregate-level utilities by a Poisson regression with fixed effects. For a regressor in the Poisson regressions, we make a set of polynomials for some degree from X^U_{sht} and X^V_{sht}, which is denoted by X^{poly}_{sht}. We model the aggregate-level utilities as follows:

$$U_{sht} = X^{\text{poly}\prime}_{sht}\beta^{\text{poly}}_U,\ V_{sht} = X^{\text{poly}\prime}_{sht}\beta^{\text{poly}}_V.$$

We use $\hat{\beta}^{\text{poly}}_U$ and $\hat{\beta}^{\text{poly}}_V$ as the estimated coefficients attached with the polynomials. And we define the estimated aggregate-level utilities by $\hat{U}_{sht} \equiv X^{\text{poly}\prime}_{sht}\hat{\beta}^{\text{poly}}_U$ and $\hat{V}_{sht} \equiv X^{\text{poly}\prime}_{sht}\hat{\beta}^{\text{poly}}_V$. For the details of this estimator, the readers can refer Galichon and Salanié (2021*b*).

5.2.2 Second step

When we directly observe the values of ι_{ht} for all hospitals and periods, we can use (2) to construct an estimator of θ. By inserting the estimation results in the first step, we construct the following moment conditions for θ:

$$\sum_s \omega_{sht}\left(X^{U,\text{base}\prime}_{sht}\beta_U\right) = \sum_s \omega_{sht}\hat{U}_{sht} - \iota_{ht},\ \forall\ h,t$$
$$\sum_s \omega_{sht}\left(X^{V,\text{base}\prime}_{sht}\beta_V\right) = \sum_s \omega_{sht}\hat{V}_{sht} + \iota_{ht},\ \forall\ h,t.$$

In Appendix B, we show a Monte Carlo exercise adopting this approach to show how to recover the structural parameters.

[26] Note that our estimation target is U_{sht} and V_{sht}. The difference from the case of Galichon and Salanié (2021*a*) is that we just need one side fixed effect.

In practice, we face a measurement problem: we cannot observe the aggregate-level transfer ι_{ht}. Instead, we can only observe the realized salaries paid by hospitals every period. It is important to note that salary represents just one component of the total transfer in this market, which also includes non-monetary aspects. For example, the hospital accepts the risk of medical incidence by allowing the less-experienced medical interns to get more practice on the job. The workload in a hospital also comprises such unobserved transfers. Furthermore, we expect that the observed salaries correlate with these unobservable terms, which makes the identification more demanding.

For this problem, we introduce a measurement model to connect the observed salaries to ι_{ht}. Denoting the salary paid in a hospital h at time t by S_{ht}, we assume that both schools and hospitals have quasi-linear utilities with respect to monetary transfers:

$$\begin{aligned}
\iota_{ht} &= \gamma_{0,U} + \gamma_{1,U} S_{ht} + \psi^U_{ht} \\
-\iota_{ht} &= \gamma_{0,V} + \gamma_{1,V} S_{ht} + \psi^V_{ht}.
\end{aligned}$$

$\gamma_{1,V}$ is expected to be negative because the salary is the amount of money paid to the doctor from the hospital. ψ^U_{ht} is the unobserved transfer from the hospital to the matched doctors, and ψ^V_{ht} is the same unobserved transfer from the doctor to the hospital.

It is likely that the unobserved transfer is correlated with the observed monetary transfer. Hence we need some instrumental variables that have an influence just on the salary. As such instrumental variables, we use the characteristics of the surrounding hospitals as in Berry, Levinsohn and Pakes (1995). The rationale behind these instruments is that a hospital considers the characteristics of other hospitals when setting its salary, whereas the unobserved transfer is not known to others. In practice, we use only the characteristics of nearby hospitals located within a 20 km radius of a given hospital as instrumental variables for the salary, even though our model accounts for all hospitals operating within the same market.

By combining the moment conditions and the measurement model, estimating equations are specificed as follows:

$$\begin{aligned}
\sum_s \omega_{sht} \hat{U}_{sht} &= \gamma_{0,U} + \gamma_{1,U} S_{ht} + \sum_s \omega_{sht} \left(X^{U\prime}_{sht} \beta_U \right) + \psi^U_{ht} \\
\sum_s \omega_{sht} \hat{V}_{sht} &= \gamma_{0,V} + \gamma_{1,V} S_{ht} + \sum_s \omega_{sht} \left(X^{V\prime}_{sht} \beta_V \right) + \psi^V_{ht}.
\end{aligned}$$

When we take the weighted average of every variable in X^U_{sht} and X^V_{sht} as independent variables in the right hand side, the above equations are just linear equations in θ. We estimate these linear equations using the instrumental variables discussed above.

6 Empirical Results

In this section, we show the estimation results. In Section 6.1, we show the estimation results of our first step. In Section 6.2, we show the estimation results of our second step: the preference parameters of both sides of the market.

6.1 First Step

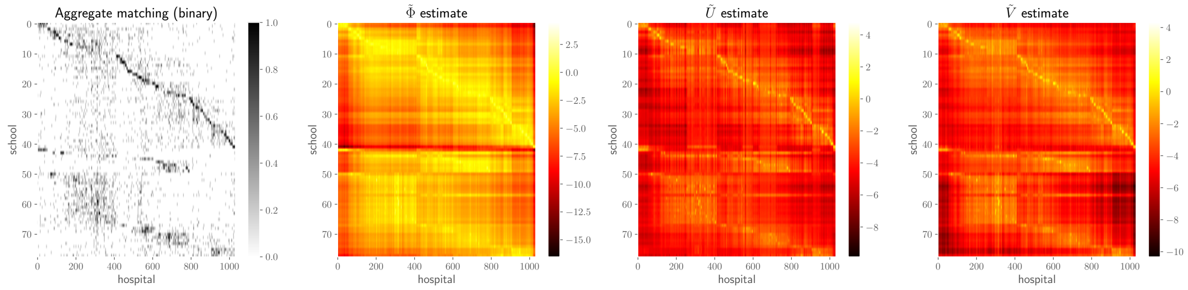
From 2017 to 2019, we estimate aggregate-level utility of both sides separately. The degree of polynomial approximating the aggregate-level utility is our tuning parameter. In this section we show the results obtained when we choose three as the degree of polynomials as this choice allows the more flexible functional form. In Appendix C.3, we show the results obtained when the degree of polynomials is set to two.

Each panel in Figure 3 shows the observed matching pattern (leftmost figure), the estimated aggregate-level social surpluses (second from the left), the estimated aggregate-level utilities from the doctors' side (second from the right), and the estimated aggregate-level utilities from the hospitals' side (rightmost figure). All figures are heatmaps in which the vertical axis represents the indices of medical schools and the horizontal axis represents the indices of hospitals.[27] In the right three heatmaps, brighter colors indicate higher values.

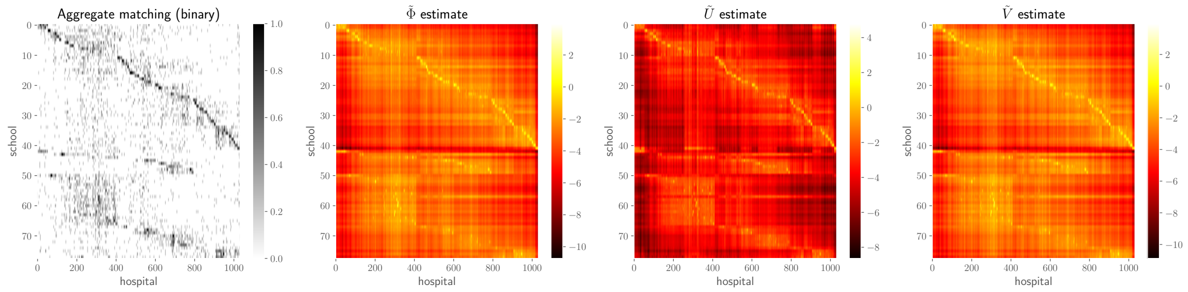
The visible pattern in the aggregate matching is well captured by our estimation. Specifically, all the heatmaps reflect the likelihood of matches between graduates from local public universities and nearby hospitals. As illustrated in the rightmost figures, even from the hospitals' perspective, graduates from closer medical schools provide higher aggregate-level utility. [28] This pattern can be interpreted as evidence of the importance of local knowledge: knowledge of the local medical environment is so critical that hospitals prefer to hire local doctors.

[27]The way to set the index is described in Section 2.

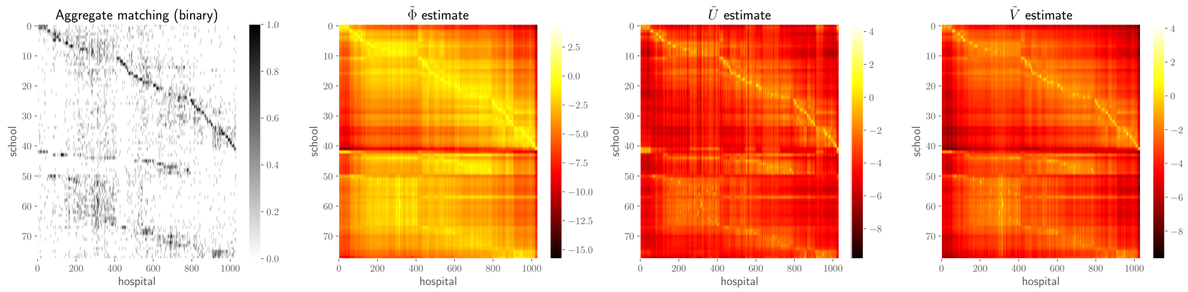
[28]In contrast, Agarwal (2015) highlights that distance influences doctors' decisions but does not account for preference heterogeneity concerning distance on the hospital side.



(a) 2017



(b) 2018



(c) 2019

Figure 3. Aggregate Matchings, Aggregate-level Utilities and Social Surpluses.

6.2 Second Step

Given the estimation results in the first step, we estimate the marginal effects of covariates on the base utilities. We include the following hospital-specific variables as the characteristics in the preference of doctor side: *logarithm of number of beds of a hospital*, which acts as the measure of the size and the quality of the hospital, *dummy variable of university hospital*, *dummy variable of governmental hospital*, *dummy variable of urban area*, and *dummy variable of Tokyo.*[29] Furthermore, we include the following pairwise variables: *logarithm of distance*, *logarithm of number of previous matches*, and *dummy variable of affiliation relationship.* As the characteristics in the preference of hospital side, in addition to the pairwise variables, we include the following university-specific variables: *dummy variable of public university*, *T score of the entrance exam*, *dummy variable of urban areas* and *dummy variable of Tokyo.*

The estimation results are shown in Table 5. Column 1 and 3 correspond to the case of OLS. Column 2 and 4 are the results obtained when we use BLP instruments for salary. The direction of the estimated coefficients of salary are alined with the expected signs when we use instruments. We adopt the results obtained using IV estimations as our main estimation results. Although we cannot reject the null hypothesis that $|\gamma_{1,U}| = |\gamma_{1,V}|$ at the 5% significance level, we use different coefficients in the subsequent counterfactual analysis rather than assume these two are equal.[30]

Distance between university and hospital negatively influences on both of the preferences of doctors and hospitals, which is aligned with the estimation results of the first step. Furthermore, as intuitive and anecdotally validated, the previous number of matches have the strong influence on the preferences. The more previous matches lowers the hurdle to apply for doctor side and the uncertainty about the quality is cleared from the point of view of hospital side. The quality measure for both sides are also impactful. From the doctor side, the number of beds of a hospital increases the utility obtained when matching with a hospital. And the hospital prefers the doctor from a public university, which is more difficult to enter. We find positive coefficients of Tokyo dummy and urban dummy in the preference of doctor side, and positive coefficient of urban dummy on hospital side.[31]

[29] Tokyo is by far the largest metropolitan area compared to other urban regions and holds a unique status as the capital, which is why we included a dedicated dummy variable for it.

[30] The p-value of the test is 0.22.

[31] If there is an implicit tax on the prefectures in the urban areas, this coefficient might be under

Table 5. Estimation Result: Preference Parameters
Degree of Polynomials = 3

	(1) University	(2) University (IV)	(3) Hospital	(4) Hospital (IV)
Constant	-5.494***	-6.724***	1.194	1.954**
	(0.187)	(0.327)	(0.776)	(0.874)
Salary (million Yen)	0.574***	2.527***	0.634***	-1.780**
	(0.128)	(0.479)	(0.146)	(0.792)
Tokyo	0.0371	0.112**	0.0251	-0.0940
	(0.0376)	(0.0461)	(0.0618)	(0.0712)
urban	-0.0307	0.0572*	0.205***	0.125***
	(0.0264)	(0.0333)	(0.0346)	(0.0447)
log(Distance)	-0.438***	-0.436***	-0.409***	-0.373***
	(0.0150)	(0.0121)	(0.0194)	(0.0217)
log(Previous Match)	1.245***	1.229***	1.551***	1.560***
	(0.0305)	(0.0231)	(0.0411)	(0.0463)
Affiliation	0.380**	0.460***	-2.007***	-2.197***
	(0.166)	(0.111)	(0.186)	(0.205)
University hospital	-0.124*	-0.0127		
	(0.0726)	(0.0788)		
Govermental hospital	0.0723**	0.0132		
	(0.0298)	(0.0330)		
log(Beds)	0.744***	0.814***		
	(0.0303)	(0.0353)		
Public university			0.287***	0.289***
			(0.0569)	(0.0610)
Prestige			-1.660**	-2.774***
			(0.734)	(0.810)
N	2847	2627	2847	2627

Standard errors in parentheses
* p<0.1, ** p<0.05, *** p<0.01

Table 6. University Preference Parameters (Unit: Million Yen)
Degree of Polynomials = 3

	(1)	(2)	(3)
Coefficient of Salary =	2.527	2.412	2.519
log(Distance)	-0.173***	-0.181***	-0.173***
	(0.03)	(0.03)	(0.03)
log(Previous Match)	0.486***	0.508***	0.487***
	(0.09)	(0.10)	(0.09)
Affiliation	0.182***	0.186**	0.182**
	(0.06)	(0.06)	(0.06)
University Hospital	-0.005	-0.001	-0.005
	(0.03)	(0.03)	(0.03)
Governmental Hospital	0.005	0.005	0.005
	(0.01)	(0.01)	(0.01)
log(Beds)	0.322***	0.338***	0.324***
	(0.06)	(0.06)	(0.06)
N	2627	2627	2627
Urban × Year		√	√
Tokyo × Year			√

Standard errors in parentheses
* p<0.1, ** p<0.05, *** p<0.01

Table 7. Hospital Preference Parameters (Unit: Million Yen)
Degree of Polynomials = 3

	(1)	(2)	(3)
Coefficient of Salary =	1.780	1.579	1.810
log(Distance)	-0.209*	-0.236*	-0.206*
	(0.10)	(0.11)	(0.10)
log(Previous Match)	0.877*	0.989*	0.862*
	(0.39)	(0.44)	(0.38)
Affiliation	-1.235*	-1.383*	-1.215*
	(0.52)	(0.59)	(0.50)
Public University	0.163*	0.188*	0.160*
	(0.08)	(0.09)	(0.07)
Prestige	-1.558*	-1.764*	-1.534*
	(0.66)	(0.74)	(0.64)
N	2627	2627	2627
Urban × Year		√	√
Tokyo × Year			√

Standard errors in parentheses
* p<0.1, ** p<0.05, *** p<0.01

Next, we evaluate the marginal effects in monetary unit. For this purpose, we take the fractions of the estimated coefficients of the covariates with respect to the coefficients of salary. The fractions and the standard errors for doctor side are shown in Table 6 and the same ones for hospital side are shown in Table 7. In both tables, we present results for three specifications that differ in whether they include interaction terms between year dummies and the urban and Tokyo dummy variables. The results remain qualitatively robust across these specifications.

For doctor side, the estimates tell that the match with a hospital which is 10% faraway decreases the utility by from 0.017 to 0.018 million yen: this is about \$108. The number of previous matches, the affiliation relationship and the number of beds of a hospital play the positive influences: 10% increase in the number of previous matches improves the utility by from 0.049 to 0.051 million yes, which is about \$319, the hiring by affiliated hospitals increases the utility by from 0.182 to 0.184 million yen, which is about $\$1,169$, and 10% increase in the number of beds imrpoves the utility by from 0.032 to 0.034

estimated. In Appendix C.1, we exploit the tightening regional caps to check this existence and conclude that the current market does not face such implicit taxation.

million yen, which is about \$210.

For hospital side, distance and the previous number of matches play the similar roles: the doctors from 10% faraway univeristy decreaes the utility of hospital by from 0.021 to 0.024 million yen, which is about \$146, and 10% increase in the number of previous matches improves the utility of hospital by from 0.086 to 0.099 million yes, which is about from \$549 to \$632. The indicator of public university also has positive impact as expected: the premium of gradtuating from a public university is from 0.160 to 0.188 million yen, which is about from $\$1,022$ to $\$1,201$.

These marginal effects of the covariates are significant even compared to the aggregate-level utilities. In Appendix C.2, we calculate the ratio of the estimated marginal effects of the covariates to the aggregate-level utility. For many covariates, a 10% change in the covariates corresponds to approximately $1 \sim 5\%$ of the aggregate-level utility on both sides.

7 Counterfactual Simulations

We simulate matching outcomes under several scenarios to evaluate the (in)efficiency of cap-based regulations in the residency matching market. For such simulations, we need to set *true capacity*, originally set by each hospital. This is because, as we explain in Section 1.1, each hospital's observed capacity is reduced so that the sum of them within a prefecture satisfies the regional cap. Since there is no formal formula to recover the true capacities, we set each hospital's true capacity to the maximum number of positions reported to the JRMP between 2015 and 2023.

We conduct two types of simulations. In Section 7.1, we compare the market outcomes between various types of aggregate equilibria and the efficient aggregate equilibrium. This analysis highlights the welfare losses caused by cap-based regulations and identifies their sources as the decrease in the number of matches in urban counties. In Section 7.2, we simulate individual preferences and examine market outcomes using practical mechanisms. This is the first empirical evaluation of the efficiency of flexible DA, and we emphasize that monetary intervention remains crucial for addressing distributional imbalances.

Table 8. Comparison between Aggregate-level Equilibria in 2017

Equilibrium	**AE**	**AE**	**EAE**
Capacity	Reduced	True	True
Floor	No	No	Yes
Match rate	0.868	0.912	0.912
Doctors' welfare	82874.9	84507.3	84514.3
Hospitals' welfare	51788.4	56209.0	56211.2
Government's revenue	0.0	0.0	$[-10.5, -7.4]$
Total welfare	134663.3	140716.3	$[140715.0, 140718.1]$
#(subsidized regions)	0	0	3
Average subsidy	0.000	0.000	-0.040
#(constraint violations)	0	3	0

* All welfare and revenue figures are expressed in units of 1 million JPY per month. Government revenue is positive when taxes are imposed on doctors and hospitals and negative when subsidies are provided. Doctors' and hospitals' welfare are scaled according to specification (1) in Table 6 and Table 7. We present the bounds of the government's net revenue, scaled by the coefficients on the doctor side and the hospital side, respectively. The total welfare is the sum of doctors' welfare, hospitals' welfare, and the government's revenue. #(constraint violations) counts the number of prefectures violating the lower bounds (among the 15 rural regions).

7.1 Inefficiency of cap-based regulations

We compare three scenarios. The first is the aggregate equilibrium under the artificially reduced capacities, which is expected to approximate the actual matching outcomes. The second scenario considers the aggregate equilibrium under the true capacity. Comparing this with the first scenario allows us to evaluate *how much welfare is lost due to the reduction in caps in urban counties* and *how many floor conditions are broken.* The third scenario examines the efficient aggregate equilibrium under true capacity and floors on rural counties. By comparing this with the second scenario, the last one clarifies *how severely the floor conditions are broken* and *how efficient the optimal monetary intervention is.*

For the third scenario, we define *rural counties* as the 15 prefectures with the lowest observed match rates from 2017 to 2019.[32] For these rural counties, floor constraints are set so that each county has at least as many residents as in the first scenario.

The simulation results for the year of 2017 are presented in Table 8. The results of the other years are shown in Appendix C.4 and there is no qualitative difference. All the values except match rates, #(subsidized regions), and #(constraint violations) are

[32]The match rate for each county is calculated as the total number of matches in the county over the three years divided by the sum of the county's total capacities across those three years.

expressed in units of 1 million JPY per month, transformed by the salary coefficients of specification (1) in Table 6 and Table 7. Note that the government's revenue varies depending on the proportion of taxes and subsidies collected from doctors versus hospitals; therefore, we display both the upper and lower bounds in the table for the case of EAE. The total welfare is the sum of the doctors' welfare, the hospitals' welfare, and the government's revenue: then we similarly display both the upper and lower bounds of the value.

We observe that relaxing the caps in urban counties increases the match rate by 5 percentage points. This relaxation also leads to an estimated increase of approximately 5 billion yen in total welfare. This welfare gain comes at the cost of three rural prefectures violating floor conditions. Nevertheless, these violations are relatively minor: as shown in the last column, under EAE, the floor conditions are breached by no more than 18.9 million yen per month. This amount is significantly lower than the welfare gains achieved.

In order to clarify the source of the welfare gain under the EAE, Figure 4 illustrates the difference in social welfare between EAE and AE across prefectures. Welfare improves across all prefectures under EAE, with particularly notable gains in urban areas—the prefectures currently subject to regional caps under the JRMP. This further suggests that a significant portion of the inefficiency in the current JRMP policy stems from overly strict caps on urban areas.

7.2 Evaluation of flexible deferred acceptance algorithm

We use the estimation results to simulate the matching outcomes under variants of deferred acceptance algorithms, which requires simulated preference lists of both sides. First, we explain the bechmark scenario which approximates the actual matching outcome. We refer to this case as first scenario. Using the estimated aggregate-level utilities, U and V, we calculate the utility of doctor i when matched with slot j as $u_{ij} = U_{s(i),h(j)} + \varepsilon_{i,h(j)}$, where $\varepsilon_{i,h(j)}$ is a random logit error. The hospital-side utility is computed similarly. Based on these simulated utilities, we construct individual preference lists and run the Deferred Acceptance algorithm with JRMP caps. The results are presented in the leftmost column of Table 9. This scenario serves as a reference point for evaluating the subsequent simulation results.

As in Section 7.1, we introduce two counterfactual scenarios: one with relaxed caps in urban areas, which is referred to as second scenario, and another with floor conditions

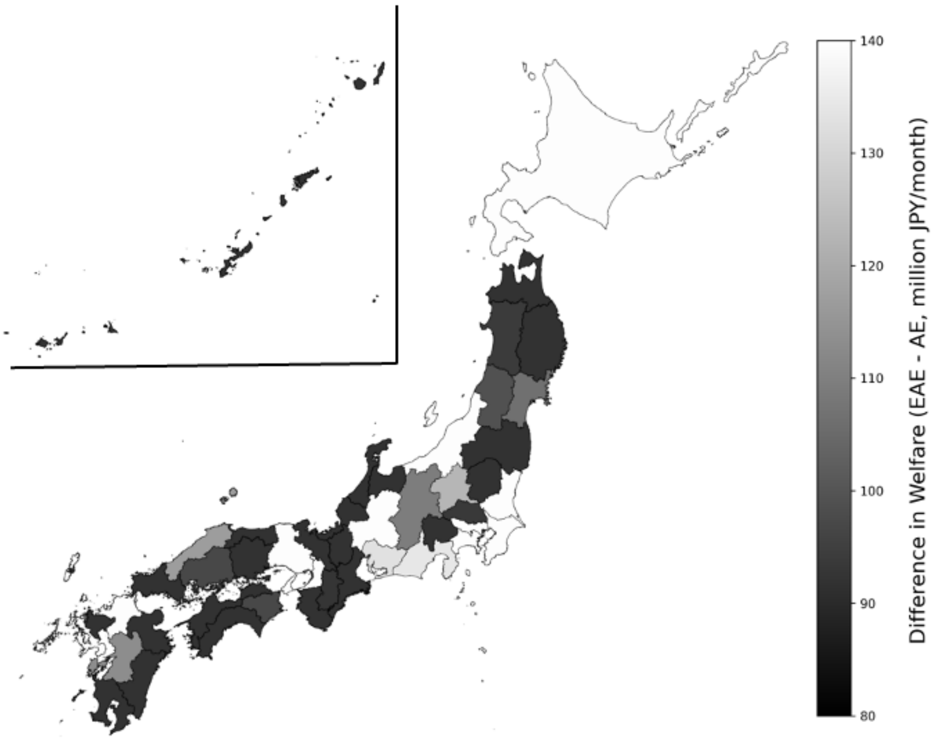


Figure 4. Difference in Welfare between EAE and AE by Prefecture

in rural areas combined with relaxed caps in urban areas, which is referred to as third scenario. Relaxing caps and imposing floor conditions affect the aggregate-level utilities on both sides through salary adjustments. Therefore, for these simulations, we first compute the aggregate-level utilities by solving the primal problem outlined in Section 4 based on the estimated baseline utilities. Specifically, in the second scenario, we compute the aggregate equilibrium, while in the third, we compute the efficient aggregate equilibrium under floor conditions. Using these aggregate-level utilities, we then run the Deferred Acceptance algorithm under the true capacities using simulated preference lists. The floors used in the thrid scenario above are set so that each prefecture has at least as many residents as in the first scenario. We again use the same 15 prefectures as rural areas where floor condtions are required.

The results are presented in the second and third columns of Table 9. The comparison among the first three scenarios aligns with the findings based on aggregate equilibria shown in Table 8. Specifically, we observe an increase in the match rate when caps in urban areas are relaxed, and total welfare rises even under the optimal subsidy.[33] These

[33]Even when using aggregate-level utilities from EAE, some prefectures fail to meet the floor conditions. This occurs due to individual-level variance arising from the finite number of agents in the market.

Table 9. Welfare Comparison of the Simulated Matchings in 2017

Scenario	**(1)**	**(2)**	**(3)**	**(4)**
Algorithm	DA	DA	DA	FDA
Capacity	Reduced	True	True	True
Floor	No	No	Yes	Yes
Match rate	0.822	0.862	0.862	0.826
Doctors' welfare	35506.0	37695.7	37715.0	35944.2
Hospitals' welfare	24674.7	25269.9	25264.3	25852.4
Government's revenue	0.0	0.0	$[-19.7, -13.9]$	0.0
Total welfare	60180.7	62965.6	$[62959.6, 62965.4]$	61796.6
#regions violating lower bounds	0	3	2	1
#doctors required to meet lower bounds	0	6	2	1
#doctors matched with urban hospitals	3258	3312	3310	2588
#doctors matched with rural hospitals	1235	1329	1335	1441
Urban hospitals' welfare	5137.0	4557.6	4544.1	4302.1
Rural hospitals' welfare	1365.4	1343.2	1364.7	1446.3

* All welfare and revenue figures are expressed in units of 1 million JPY per month. Government revenue is positive when taxes are imposed on doctors and hospitals and negative when subsidies are provided. Doctors' and hospitals' welfare are scaled according to specification (1) in Table 6 and Table 7. We present the bounds of the government's net revenue, scaled by the coefficients on the doctor side and the hospital side, respectively. The total welfare is the sum of doctors' welfare, hospitals' welfare, and the government's revenue. "#regions violating lower bounds" indicates the number of prefectures whose matched doctor count is less than that of scenario (1). "doctors required to meet lower bounds" indicates how many additional doctors must be matched with rural hospitals so that these prefectures exceed the matched doctor count in scenario (1).

consitency validates the correspondence between the aggregate-level analysis and the individual-level analysis where we use mechanism to compute the market outcome.

Finally, we apply flexible DA to this market, which is refered to as scenario 4. For this scenatio, we use the same aggregate-level utilities as in scenario 3 and simulate the preference lists. For the floor conditions, we use the number of matches in scenario 3. We search the minimum reduction ratio of total capacities in urban areas under which the floor conditions of rural prefectures are satisfied.[34] The fourth column in Table 9 shows the market outcomes of this setting: where we reduce 34% of true capacities in urban areas. Even under this reduction, the match rate increases by 0.4 percentage point and the total welfare increases from scenario 1. As expected, the hospital welfare and number of matches in urban areas decreases compared to scenario 2. This decrease overwhelms the welfare gain in rural areas and the total welfare is less than scenario 3. Given flexible

[34]We set the upper bounds on six urban regions (Tokyo, Kanagawa, Aichi, Kyoto, Osaka, and Fukuoka) as α% of the total true capacities in each prefecture. We choose the maximum α that the output matching satisfies the floor conditions in rural areas.

DA is the latest mechanism exploiting the slots in urban areas in an efficienct way, from this observation we conclude that cap-based regulations are not so effective in modifying the distributional imbalances. Instead, even small amount of monetary intervention can work to make inflows into such under represented areas.

8 Conclusion

In this study, we propose a theoretical and empirical framework to evaluate the efficiency of cap-based regulations in matching markets. First, we incorporate regional constraints, including caps and floors, into the transferable utility matching model. Next, we demonstrate that, under certain assumptions, the same data-generating process can be constructed using only aggregate-level objects. This extension enables us to identify the model primitives and simulate various market outcomes. A key innovation in our empirical strategy is the use of salary as the observed component of the transfer between matched agents, rather than merely as a proxy for the transfer.

The estimation results reveal that both sides of the market exhibit horizontal differentiation. Specifically, the distance and the past number of matches between medical schools and hospitals influence the preferences on both sides. Building on these findings, we simulate a counterfactual matching market scenario in which the government removes the current caps on matches in urban counties. Under this scenario, only three rural prefectures fail to meet their floor conditions. Moreover, by implementing the optimal subsidy for these rural areas, the government can still achieve welfare gains while ensuring that the floor conditions are satisfied. Finally, we examine the effectiveness of the flexible DA algorithm in addressing distributional imbalances. Our results indicate that while it improves welfare in urban counties, achieving the floor conditions in rural counties requires a 34% reduction in urban hospital capacities which results in huge welfare loss. We conclude that monetary intervention by the government, combined with the flexible DA algorithm, effectively addresses distributional imbalances and enhances overall welfare.

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A Omitted Proofs

A.1 Proof of Lemma 1

First, we will show the following lemma: [35]

Lemma 2. *Suppose that U and V are aggregate-level utilities given (u, v) (i.e., (4) holds). Suppose that $(d, (u, v))$ is a stable outcome. Then, we have $U_{xy} + V_{xy} \geq \Phi_{xy} + w_{z(y)}$ with equality when $\mu_{xy} > 0$ for each $(x, y) \in T$.*

Proof. Fix any $(x, y) \in T$. First, we show $\Phi_{xy} - w_{z(y)} \leq U_{xy} + V_{xy}$. Suppose that $i \in x$ is matched with hospital $j \in y$. We have

$$\begin{aligned}
u_i &= \max_{j \in J} \{\tilde{\Phi}_{ij} - v_j\} \\
&= \max_{y \in Y} \max_{j \in Y} \{\Phi_{ij} - w_{z(y)} - v_j\} \\
&= \max_{y \in Y} \max_{j \in Y} \{\Phi_{xy} - w_{z(y)} + \varepsilon_{iy} + \eta_{xj} - w_{z(y)} - v_j\} \\
&= \max_{y \in Y} \{\Phi_{xy} - w_{z(y)} + \varepsilon_{iy} + \max_{j \in y} \{\eta_{xj} - v_j\}\} \\
&= \max_{y \in Y} \{\Phi_{xy} - w_{z(y)} + \varepsilon_{iy} - V_{xy}\}
\end{aligned}$$

Thus, for any $i \in x$, we have

$$\begin{aligned}
u_i &= \max \left\{ \max_{y \in Y} \{\Phi_{xy} - w_{z(y)} + \varepsilon_{iy} - V_{xy}\}, \varepsilon_{i,y_0} \right\} \\
&= \max_{y \in Y_0} \{\Phi_{xy} - w_{z(y)} + \varepsilon_{iy} - V_{xy}\}.
\end{aligned}$$

Hence,

$$\Phi_{xy} - w_{z(y)} \leq u_i - \varepsilon_{iy} + V_{xy}.$$

[35] The following proof of Lemma 2 is almost identical to the proof of Proposition 1 of Galichon and Salanié (2021*a*).

By taking the infimum over $i \in x$, we have

$$\Phi_{xy} - w_{z(y)} \leq U_{xy} + V_{xy},$$

for each $x \in X$ and $y \in Y$.

Next, suppose that $\mu_{xy} > 0$. This implies that there exist $i \in x$ and $j \in y$ such that $d_{ij} = 1$. For this pair, we have $u_i + v_j = \Phi_{ij} - w_{z(y)}$. Suppose toward contradiction that $U_{xy} + V_{xy} > \Phi_{xy}$. By (4), we have $\Phi_{xy} - w_{z(y)} < u_i - \varepsilon_{iy} + v_j - \eta_{xj}$, and thus $\Phi_{ij} - w_{z(y)} < u_i + v_j$. A contradiction.

□

Proof of Lemma 1. Fix any type $x \in X$ and doctor $i \in x$. By definition of U_{xy}, we have

$$\begin{aligned}
& U_{xy} \leq u_i - \varepsilon_{iy}, \ \forall y \in Y_0 \\
\iff \ & u_i \geq U_{xy} + \varepsilon_{iy}, \ \forall y \in Y_0 \\
\iff \ & u_i \geq \max_{y \in Y_0} \{U_{xy} + \varepsilon_{iy}\}.
\end{aligned}$$

Similarly, for any type $y \in Y$ and doctor $j \in J$ with type y, we have $v_j \geq \max_{x \in X_0}\{V_{xy} + \eta_{xj}\}$.

We want to claim that $u_i \leq \max_{y \in Y_0}\{U_{xy} + \varepsilon_{iy}\}$. Suppose toward contradiction that there exists type $x \in X$ and doctor $i \in x$ such that

$$u_i > \max_{y \in Y_0} \{U_{xy} + \varepsilon_{iy}\}.$$

First, consider the case where i is matched with some hospital $j \in y$. Then

$$\begin{aligned}
\Phi_{ij} - w_{z(y)} &= u_i + v_j \\
&> \left(\max_{y' \in Y_0} \ U_{xy'} + \varepsilon_{iy'} \right) + \left(\max_{x' \in X_0} \ V_{x'y} + \eta_{x'j} \right) \\
&\geq U_{xy(j)} + \varepsilon_{iy(j)} + V_{xy(j)} + \eta_{xj} \\
&\geq \Phi_{xy} - w_{z(y)} + \varepsilon_{iy(j)} + \eta_{xj} \quad (\because \text{ Lemma 2}) \\
&= \Phi_{ij} - w_{z(y)}.
\end{aligned}$$

A contradiction. Next, consider the case where i is unmatched. Then

$$u_i = \Phi_{i,y_0} = \varepsilon_{i,y_0} > \max_{y \in Y_0}\{U_{xy} + \varepsilon_{iy}\} \geq \varepsilon_{i,y_0}.$$

A contradiction. Therefore, we have $u_i \leq \max_{y \in Y_0}\{U_{xy}+\varepsilon_{iy}\}$ and hence $u_i = \max_{y \in Y_0}\{U_{xy}+\varepsilon_{iy}\}$. We can show $v_j = \max_{x \in X_0}\{V_{xy} + \eta_{xj}\}$ in a similar manner. $\square$

A.2 Proof of the Uniqueness of the Solutions of (P) and (D)

We will show the following:

Lemma 3. *If G and H are strictly convex and differentiable,*[36] *(P) and (D) have unique solutions for any Φ.*

For (P), since G and H are differentiable (WDZ theorem), G^* and H^* are strictly convex (Proposition D.14 of Galichon (2018)). Therefore, the objective function of (P) is strictly convex in μ, which implies the uniqueness of the solution.

For (D), suppose toward a contradiction that there are two different optimal solutions $\alpha := (U, V, \bar{w}, \underline{w})$ and $\beta := (U', V', \bar{w}', \underline{w}')$. Note that $\gamma := \frac{1}{2}(\alpha + \beta)$ is also feasible. If either $U \neq U'$ or $V \neq V'$, then γ gives a strictly lower value due to the strict convexity of G and H, which contradicts the optimality of α and β.

Suppose that $U = U$ and $V = V'$. We must have $(\bar{w}, \underline{w}) \neq (\bar{w}', \underline{w}')$. Since G is strictly convex, we have $\frac{\partial G}{\partial U_{xy}} > 0$ for each (x, y). By the complementary slackness condition with respect to U_{xy}, we have $U_{xy} + V_{xy} = \Phi_{xy} + \bar{w}_{z(y)} - \underline{w}_{z(y)}$ and $U'_{xy} + V'_{xy} = \Phi_{xy} + \bar{w}'_{z(y)} - \underline{w}'_{z(y)}$ for each (x, y). Since $U = U'$ and $V = V'$, this implies that

$$\bar{w}_z - \underline{w}_z = \bar{w}'_z - \underline{w}'_z.$$

for each z. Since $\bar{o}_z > \underline{o}_z$, we must have $\bar{w}_z \underline{w}_z = 0$; otherwise, there exists $\varepsilon > 0$ such that $(U, V, \tilde{\bar{w}}, \underline{\tilde{w}})$, where $\tilde{\bar{w}}_z := \bar{w}_z - \varepsilon$ and $\underline{\tilde{w}}_z := \underline{w}_z - \varepsilon$ attains a strictly lower value. Similarly, we have $\bar{w}'_z \underline{w}'_z = 0$. However, these combined with (A.2) imply that $\bar{w} = \bar{w}'$ and $\underline{w} = \underline{w}'$: if $\bar{w}_z > 0$, then $\underline{w}_z = 0$, $\underline{w}'_z = 0$, and $\bar{w}_z = \bar{w}'_z$. If $\underline{w}_z > 0$, then $\bar{w}_z = 0$, $\bar{w}'_z = 0$, and $\underline{w}_z = \underline{w}'_z$. If $\bar{w}_z = \underline{w}_z = 0$, then $\bar{w}'_z = \underline{w}'_z = 0$. A contradiction.

[36] This holds under Assumptions 1-5.

A.3 Proof of Theorem 1

First, we show two lemmas used in the main proof.

Lemma 4. *Under Assumptions 1-5, G and H are strictly increasing and strictly convex.*

Proof. $\underline{G \text{ is strictly increasing.}}$ Take any $U^1, U^2 \in \mathbb{R}^{N\times M}$ such that $U^1 \geq U^2$ and $U^1 \neq U^2$. Then $G(U^1) \geq G(U^2)$ by definition. In addition, note that $U^1_{xy} > U^2_{xy}$ holds for some $x \in X$ and $y \in Y$. Since P_x has full support, we have

$$\Pr_{\varepsilon_i}(u_i = U^1_{xy} + \varepsilon_{iy}) \geq \Pr_{\varepsilon_i}(u_i = U^2_{xy} + \varepsilon_{iy}) > 0.$$

Because $\mathbb{E}_{\varepsilon_i}\left[u_i \mid u_i = U_{xy} + \varepsilon_{iy}\right]$ is strictly increasing in U_{xy}, we have

$$\begin{aligned}\mathbb{E}_{\varepsilon_i}\Big[u_i \mid u_i = U^1_{xy} + \varepsilon_{iy}\Big] \cdot \Pr_{\varepsilon_i}(u_i = U^1_{xy} + \varepsilon_{iy}) \\ > \mathbb{E}_{\varepsilon_i}\Big[u_i \mid u_i = U^2_{xy} + \varepsilon_{iy}\Big] \cdot \Pr_{\eta_j}(u_i = U^2_{xy} + \varepsilon_{iy}),\end{aligned}$$

and thus $G(U^1) > G(U^2)$ holds.
$\underline{G \text{ is strictly convex.}}$ Take any $U^1, U^2 \in \mathbb{R}^{N\times M}$ and $s \in [0, 1]$. Since

$$\begin{aligned} sG(U^1) + (1-s)G(U^2) &= \sum_x n_x \mathbb{E}\left[\Big(\max_y\ s(U^1_{xy} + \varepsilon_{iy})\Big) + \Big(\max_y\ (1-s)(U^2_{xy} + \varepsilon_{iy})\Big)\right] \\ &\geq \sum_x n_x \mathbb{E}\left[\max_y\ sU^1_{xy} + (1-s)U^2_{xy} + \varepsilon_{iy}\right] \\ &= G\Big(sU^1 + (1-s)U^2\Big)\end{aligned}$$

holds, G is a convex function.

Now suppose $U^1 \neq U^2$. Then $U^1_{xy} \neq U^2_{xy}$ holds for some $x \in X$, $y \in Y$. Without loss of generality, assume $U^1_{xy} > U^2_{xy}$. Since P_x is of full support,

$$\Pr\left(\left\{\varepsilon_i \colon U^1_{xy} + \varepsilon_{iy} > \max_{y'\neq y} U^1_{xy'} + \varepsilon_{iy'} \quad \wedge \quad \max_{y'\neq y} U^2_{xy'} + \varepsilon_{iy'} > U^2_{xy} + \varepsilon_{iy}\right\}\right) > 0$$

holds. This implies that

$$\Big(\max_y\ s(U^1_{xy} + \varepsilon_{iy})\Big) + \Big(\max_y\ (1-s)(U^2_{xy} + \varepsilon_{iy})\Big) > \max_y\ sU^1_{xy} + (1-s)U^2_{xy} + \varepsilon_{iy}$$

occurs with strictly positive probability, and thus (A.3) > (A.3) holds. Therefore, for any $s \in (0,1)$, we have

$$sG(U^1) + (1-s)G(U^2) > G\Big(sU^1 + (1-s)U^2\Big),$$

which implies G is strictly convex. Similarly, we can show H is also strictly increasing and strictly convex. □

Proof of Theorem 1

Part 1: Suppose that w is an optimal taxation policy. For any taxation policy w, given $(\Phi_{ij})_{ij}$ and w, a stable outcome $(d,(u,v))$ is realized, and the corresponding μ, U, and V are defined. Then, μ is a solution to (P). Since u and v are a part of the stable outcome, U and V must satisfy $U_{xy} + V_{xy} \geq \Phi_{xy} - w_{z(y)}$ for each x and y, and the market clearing condition. Since G and H are strictly increasing and convex, the market clearing condition must hold at the optimum of (D). Thus, by the uniqueness of the solution (Lemma 3), the aggregate-level utilities U and V coincide with the ones in the optimal solution to (D). Therefore, for the optimal solutions to $(U, V, \bar{w}, \underline{w})$, we have $w_z = \mathbb{1}\{\bar{w}_z > 0\}\bar{w}_z - \mathbb{1}\{\underline{w}_z > 0\}\underline{w}_z$; otherwise it violates the uniqueness of $(U, V, \bar{w}, \underline{w})$. [37]

Part 2: Suppose that μ and $(U, V, \bar{w}, \underline{w})$ are the unique optimal solutions to (P) and (D), respectively. We want to claim that w, defined as $w_z = \mathbb{1}\{\bar{w}_z > 0\}\bar{w}_z - \mathbb{1}\{\underline{w}_z > 0\}\underline{w}_z$, is an optimal taxation policy. Suppose toward contradiction that there is another optimal taxation policy w'. Let μ' and (U', V') be the aggregate-level matching and utilities under w'. Then, μ' is the solution to (P); the market clearing condition and the uniqueness of the solution imply that U' and V' is a part of the optimal solutions of (D). Then, by the same argument as Part 1, w' must satisfy $w'_z = \mathbb{1}\{\bar{w}_z > 0\}\bar{w}_z - \mathbb{1}\{\underline{w}_z > 0\}\underline{w}_z$ for each z, and the uniqueness of the solution imply $w = w'$. A contradiction. □

[37] Let $\bar{w}'_z := \mathbb{1}\{w_z > 0\}w_z$ and $\underline{w}'_z := -\mathbb{1}\{w_z < 0\}w_z$. Tuple $(U, W, \bar{w}', \underline{w}')$ is feasible in (D).

A.4 The objective function of (P) corresponds to social welfare

Recall the definition of the Legendre-Fenchel transforms:

$$G^*(\mu) := \begin{cases} \sup_U \left\{ \sum_{x \in X} \sum_{y \in Y_0} \mu_{xy} U_{xy} - G(U) \right\} & \left(\forall x \in X, \sum_{y \in Y_0} \mu_{xy} \leq n_x \right) \\ \infty & \text{otherwise} \end{cases},$$

$$H^*(\mu) := \begin{cases} \sup_V \left\{ \sum_{y \in Y} \sum_{x \in X_0} \mu_{xy} V_{xy} - H(V) \right\} & \left(\forall y \in Y,\ \sum_{x \in X_0} \mu_{xy} \leq m_y \right) \\ \infty & \text{otherwise} \end{cases}.$$

Observe that G^* and H^* are both continuous in μ on their effective domains. Suppose that (μ, U, V) forms an aggregate equilibrium. The market clearing condition states $\mu \in \partial G(U)$, where ∂G denotes the subgradient of G. By Proposition D.13 of Galichon (2018), we have $G(U) + G^*(\mu) = \sum_{x,y} \mu_{xy} U_{xy}$. Similarly, we have $H(V) + H^*(\mu) = \sum_{x,y} \mu_{xy} V_{xy}$. Thus, we have

$$\begin{aligned} \sum_{x,y} \mu_{xy} \Phi_{xy} + \mathcal{E}(\mu) &= \sum_{x,y} \mu_{xy} \Phi_{xy} + \left(G(U) + H(V) - \sum_{x,y} \mu_{xy} (U_{xy} + V_{xy}) \right) \\ &= G(U) + H(V), \end{aligned}$$

where the last equality holds since $U_{xy} + V_{xy} = \Phi_{xy}$ when $\mu_{xy} > 0$.

A.5 Another scenario for stable outcomes: deferred acceptance with endogenous wages

Consider the following dynamic game:

1. Hospitals set the amount of monetary transfer (henceforce *wage*) $t = (t_{ij})_{ij}$;

2. Doctors and hospitals submit their preference lists after observing wages t;

3. The matching is finalized by the standard deferred acceptance (DA) algorithm.

Let $\mu \colon I \cup J \to I_0 \cup J_0$ be a function such that $\mu(k)$ denotes the partner of agent k under μ. Denote the base utilities by $\tilde{u} = (u_{ij})_{ij}$ and $\tilde{v} = (v_{ij})_{ij}$ (see Section 5 for definition. Note that $\Phi_{ij} = u_{ij} + v_{ij}$.) Given t, doctor i's payoff under matching μ is $u_{i,\mu(i)} + t_{i,\mu(i)}$; job slot j's payoff is $v_{\mu(j),j} - t_{\mu(j),j}$.

We assume that, in the second stage, all agents truthfully report their preference ranking to the mechanism. Since the DA algorithm is strategy-proof for one side, this is equivalent to assuming that one of the two sides, say hospital-side, reports their preferences always truth-telling.

Fix the job-slot-optimal stable outcome $(d, (u, v))$ given Φ.[38] For (i, j) matched under d (i.e., $d_{ij} = 1$), there exists t^*_{ij} such that $u_i = u_{ij} + t^*_{ij}$ and $v_j = v_{ij} - t^*_{ij}$. For (i, j) such that $d_{ij} = 0$, choose any t^*_{ij} that satisfy $u_{ij} + t^*_{ij} \leq u_i$ and $v_{ij} - t^*_{ij} \leq v_j$. Note that such t^*_{ij} must exists since, by stability, we have $u_{ij} + v_{ij} \leq u_i + v_j$.[39] Given t^*, in the second stage, each doctor i submits a preference ranking according to $u_{ij} + t^*_{ij}$, and each job slot j submits a preference ranking according to $v_{ij} - t^*_{ij}$. By construction, each agent prefers the partner matched under d most. Hence, the following lemma holds.

Lemma 5. *For any $i \in I$ and $j \in J$,*

$$v_j = \max_{i \in I_0} \left\{ v_{ij} - t^*_{ij} \right\}, \quad u_i = \max_{j \in J_0} \left\{ u_{ij} + t^*_{ij} \right\}.$$

We also assume that, if there is a tie, each job slot always place the partner under d on the top of the preference list in the second stage.

Given this behavior in the second stage, we can show that setting t^* forms a NE in the first stage since $(d, (u, v))$ is the job-slot-optimal stable outcome.

Lemma 6. *Suppose that, in the last stage, all the agents truthfully report their preferences to the DA algorithm and breaks the tie in favor of the partner under d. Then, it forms a Nash equilibrium in the first stage that all the hospitals set t^* as their wages.*

Proof. Fix any job slot j. We will check if j can be strictly better off by choosing $(t_{ij})_i \neq (t^*_{ij})_i$. Suppose toward contradiction that it is possible under $t_j := (t_{ij})_i$. Let $i \in I_0$ denote the partner of j under d.

First, we show that j should be matched with $i' \neq i$ with t_j: this is clear if $i = i_0$. If $i \in I$ and $i' = i$, it violates the job-slot-optimality of $(d, (u, v))$.

Suppose that j is matched with $i' \neq i$ with t_{ij}. Then, we have $v_{ij} - t_{ij} > v_j$ and $u_{ij} + t_{ij} \geq u_i$.[40] However, this implies that $\Phi_{ij} = u_{ij} + v_{ij} > u_i + v_j$, which violates the stability. A contradiction. □

[38]The set of stable outcomes form a lattice. See Chapter 8 of Roth and Sotomayor (1990) for a textbook reference.

[39]$t^*_{ij} := \frac{1}{2} \left((v_{ij} - v_j) + (u_i - u_{ij}) \right)$ works, for example.

[40]This inequality is actually strict due to the tie-breaking assumption.

The outcome of the gameplay described here coincides with the stable outcome $(d,(u,v))$. Moreover, given t^* set in the first stage, no agent has incentive to deviate from the truthful report.

B Monte Carlo Simulation

We start by describing the overall setting of Monte Carlo simulation. All the detail parameter values are left to Appendix B.3. There are 10 prefectures, numbered from 0 to 9, grouped into three regions: $\{0,1\} \in R_0$, $\{2,3,4,5\} \in R_1$, and $\{6,7,8,9\} \in R_2$. R_0 represents an urban area, and R_1 and R_2 are rural areas. The government worries about the inefficient supply of medical services in R_2 and tryies to satisy a lower bound in terms of number of matches in the region.

We have 20 hospitals in total. Each hospital is placed in one of the prefectures based on a multinomial distribution. Hospital characteristics are modeled dynamically. When we denote each hospital by h, each hospital's capacity, denoted by c_{ht}, starts with a Poisson distribution at time $t = 0$ and evolves over time through a stochastic process involving increments and decrements modeled by independent Poisson distributions. We use j to denote each slot in a hospital. Other hospital-specific characteristics like the number of beds are captured by a variable z_{ht}, which follows a normal distribution.

We have 200 doctors and they are distributed among the prefectures in a similar way of the hospitals. Each doctor belongs to a one of 20 medical schools, and the schools themselves are distributed among prefectures, also based on a multinomial distribution. Each school has the equal split of the docotors in the same prefecture. The schools have characteristics like an average ability measures following a normal distribution. We use s to denote the school and i to denote a doctor.

We define the net joint surplus generated by a matching between a slot j and a doctor i at time t in the following way:

$$\Phi_{ijt} = \Phi_{sht} + \xi_{ijt},$$

where

$$\Phi_{sht} = U_{sht}^{\text{base}} + V_{sht}^{\text{base}},\ \xi_{ijt} = \varepsilon_{iht} + \eta_{sjt},$$

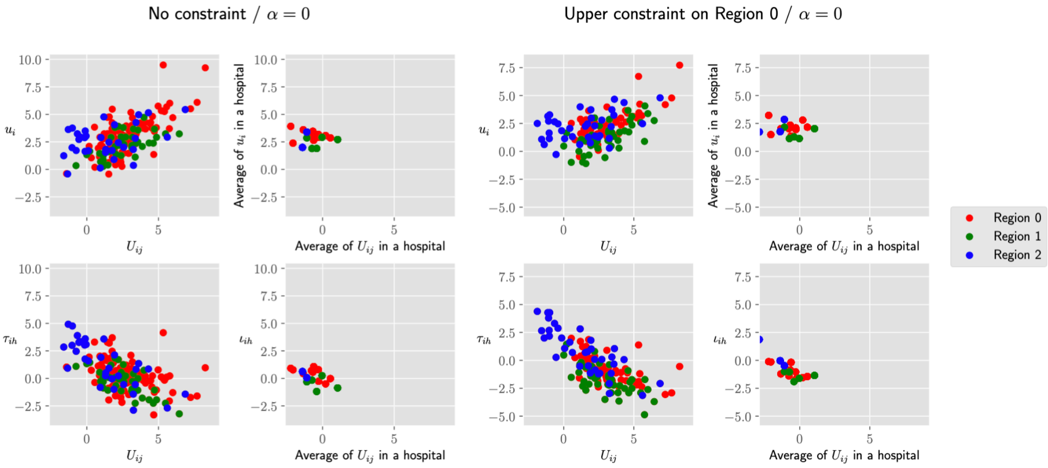


Figure 5. Simulated Stable Outcomes

and

$$\begin{aligned} U_{sht}^{\text{base}} &= \beta_{1,1} w_{1,ht} + \beta_{1,2} w_{2,ht} + \beta_2 \left| l_s - l_h \right| + \beta_3 1\{h \in R_1 \text{ or } h \in R_2\}, \\ V_{sht}^{\text{base}} &= \gamma_1 x_{1,st} + \gamma_2 x_{2,st}, \\ \varepsilon_{iht} &\sim Ex1, \ \eta_{sjt} \sim Ex1. \end{aligned}$$

$|l_s - l_h|$ is a measure of the distance between school s and hospital h: in this simulation, we define this as the absolute value of the gap between the prefecture index. And the last term in U_{sht}^{base} captures the negative impact on the utility from living in rural areas. Note that this rural areas include R_1 which is not the target of the subsidy to assure the lower bound on the matching outcomes. We also use $U_{ijt} = U_{sh(j)t}^{\text{base}} + \varepsilon_{ih(j)t}$ and $V_{ijt} = V_{s(i)ht}^{\text{base}} + \eta_{s(i)jt}$ to denote the individual level preferences.

B.1 Simulation

We compute a stable outcome of an instance of the above market at one time period. The number of matches in each region is 94, 43, and 33. The number of unmatched doctors is 30 and the number of unmatches of slots is 46. Imagine that the government set an upper bound on R_0 to increase the number of matches in rural regions. When we set the upper bound on R_0 to 60, an equilibrium number of mathces are: 60, 45, and 37. The number of unmatched doctors is 58 and the number of unmatches of slots is 74. Under

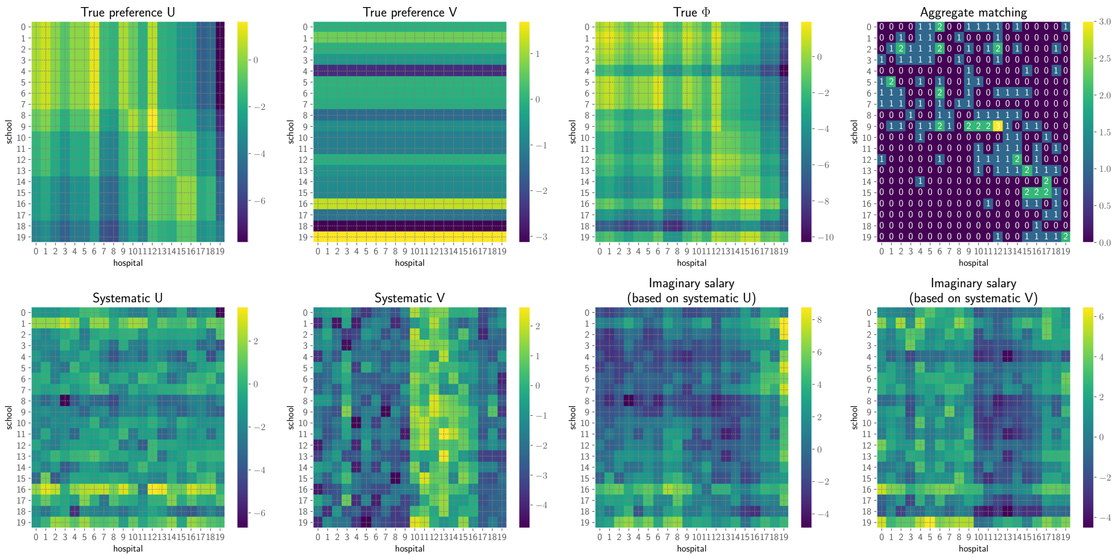


Figure 6. Aggregate Objects

this regional constraint, the tax levied on the matchings in R_0 is 3.149.

Figure 5 depicts the scatter plots of several equilibrium objects when we set $\alpha = 0$, which implies that all the amount of tax is levied on hospital side. The left four panles are obtained when we set no regional constraint, and the right four panels are obtained when we set an upper bound on R_0 to 60. The four panels in each left and right half set of panels depict the same things for the case of no constraint and regional constraint. The first and thrid columns are the results in the stable outcome (with optimal tax) where each dot represents a match. The upper panels are the scatter plots of preference of doctor i has for slot j, U_{ij}, and the utilities attained in a stable outcome, u_i. The lower panels are the scatter plots of U_{ij}^{base} and the transfer in a stable outcome, τ_{ih}. The second and fourth columns represent aggregate level objetcs: the upper panels are the scatter plots of the average of U_{ij} and u_i among a matches in a hospital, and the lower panels are the scatter plots of the average of U_{ij} and aggregate-level transfer, ι_{ih}, from a hospital. In all scatter plots, a red marker represents a match or a hospital in R_0, a green marker for R_1 and a blue market for R_2.

As expected, the utility attained in a stable outcome is higher when a doctor can be matched with a preferred slot whereas the transfer decreases. This decrease is also reflected in a decrease in aggregate-level transfer from a hospital: when the average of U_{ij} in a macthes of a hospital increases, the aggregate-level transfers from the hospital decreases. The impact of a regional constraint on the aggregate-level transfers is clear: in the constrained region, R_0, they decrease under the constraint compared with the case of no constraint. This is true in the level sense and the decrease is larger than the changes in other regions. Note that the changes in the aggregate-level transfers and their sizes depend on the value of α. For example, in the extreme case of $\alpha = 1$, the aggregate-level transfers in R_0 increases under the regional constraint. Hence, it is important to estimate the division of tax on hospital side and school side.

Hereafter, we set $\alpha = 0.2$. Figure 6 summarizes the aggregate-level objects computed based on the simulated stable outcome. In all the heatmaps, the horizontal axis represnts hospitals and the vertical axis represents schools. Aggregate matching is depicted in the upper panel in the most right column. The number annotated in each cell represents the number of matches between a hospital and a school. Aggregate-level utilities are computed following the definition stated in (4).

For the ease of argument, we name the gap between the aggregate-level utilities and the aggregate-level base utilities by *imaginary salary*: the imaginary salary from school is defined as $\chi^U_{sh} \equiv U_{sh} - U^{\text{base}}_{sh}$ and the same one from the hospital side is defined as $\chi^V_{sh} \equiv V^{\text{base}}_{sh} - V_{sh}$.[41] Th lower left two panels in Figure 6 show the imaginary salaries between schools and hospitals. The number of doctors in our simulation is 200, which is insufficient for approximating the market with infinite number of doctors. This makes the gap between the two imaginary wages computed based on U and V.[42]

B.2 Estimation

The estimation results in the first stage are depicted in Figure 7. The upper panels are the heatmap of the true values of Φ_{sh}, U_{sh}, and V_{sh}. They are the estimation targets. The lower panels are the estimation results for the corresponding upper panels. We set the degree of the polynomials to two. The estimated social surplus takes the similar patterns of the true social surplus whereas the estimated aggregate-level utilities show different patterns from the true values of them. These gaps are due to the incompleteness of polynomial approximations in equation (5.2.1). In practice, we handle this problem by inclduing non-linearly transformed base variables when making polynomial series.

For the second stage estimation, we simulate matching outcomes for two periods. In

[41] This name is from the fact that the results of the following two are the same: (1) the agents in one side chooses the agent in the other side by comparing the sum of preference term, imaginary salary, and individual disturbance and (2) Aggregate matching outcome.

[42] We can show that these two must be equal in the inifite sample case.

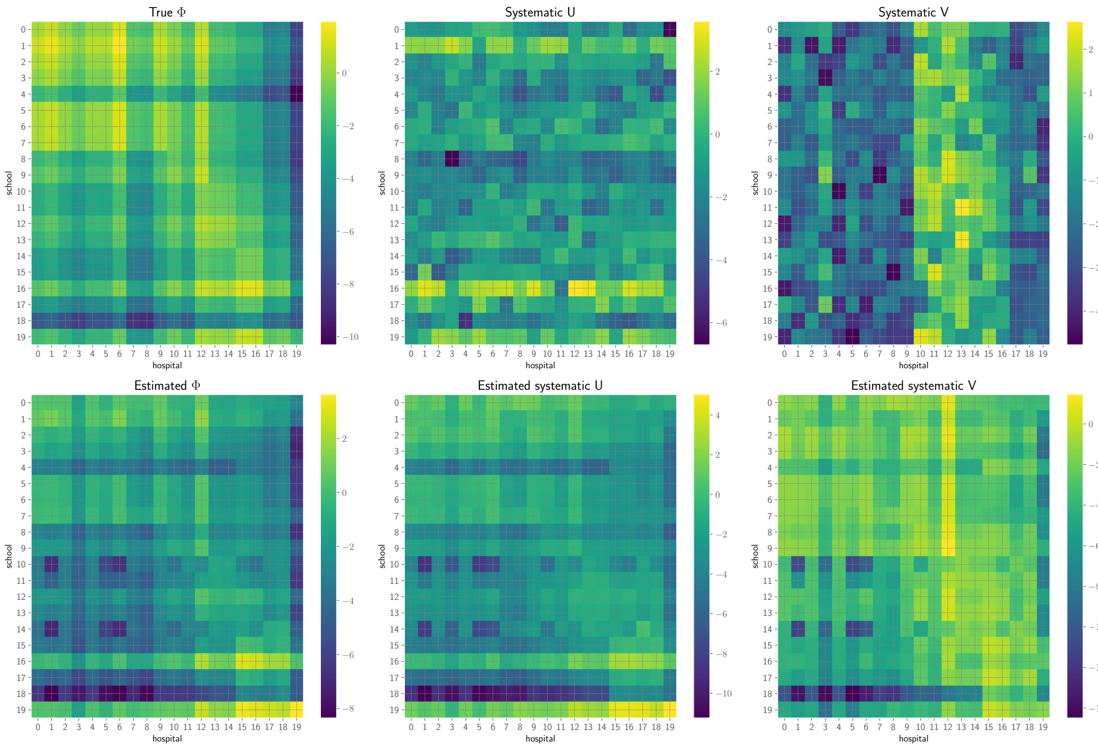


Figure 7. Estimation Results of the First Stage

Table 10. Estimates

Parameter	γ_1	γ_2	$\beta_{1,1}$	$\beta_{1,2}$	β_3	β_2	$\frac{\alpha}{T}\sum_t w_{0t}$	$\frac{1-\alpha}{T}\sum_t w_{0t}$
Estimate	0.151	-0.0384	0.907	-0.490	-0.527	-0.879	0.265	1.471
Standard Dev.	(0.178)	(0.220)	(0.0670)	(0.0496)	(0.0922)	(0.130)	(0.128)	(0.156)
True Value	1.00	-0.400	1.00	-0.500	-1.00	-1.00	0.378	1.510

the first period, the government set the upper bound on region 0 to 80 and in the second period, the upper bound is changed to 60. Because, in this exercise, we assume that the true value of aggregate-level transfers are observable, we use the moment conditions (5.2.2) directly to construct a minimum distance estimator. We leave the detail of the construction of this estimator in Appendix B.3. In this exercise, we use the time average version of the moment conditions and so the tax term is just identified as the time average of the levied tax.

Table 10 summarizes the estimation results of the second stage. The first six columns are the structural parameters in equation (B). The last two columns are the average taxes levied on docotr side and hospital side. From these estimation results, the estimate of α is 0.153 whereas the true value is 0.2. Based on these estimates, we can conduct counterfactual analysis: for example, the taxes in the alternative regional constraints, the matching outcomes, and the salaries are obtained by solving the equilibrium.

B.3 Minimum distance estimator

We use the time average of both of the sides to construct the moment conditions (5.2.2). Note that, in this case, it is impossible to idenify δ^U_{ht} and δ^V_{ht} for every t because the summation of them with respect to the time determines the moment values. Hence, all we can identify is the average tax levied through the time periods. We define a function g to represent the moment conditions:

$$g(\theta) \equiv \begin{pmatrix} \frac{1}{T}\sum_t \left(\sum_s \omega_{s1t}\left(X^{U\prime}_{s1t}\beta_U - \delta^U_{Ht}\right) - \sum_s \omega_{s1t}\hat{\hat{U}}_{s1t} - \iota_{1t}\right) \\ \vdots \\ \frac{1}{T}\sum_t \left(\sum_s \omega_{sHt}\left(X^{U\prime}_{sHt}\beta_U - \delta^U_{Ht}\right) - \sum_s \omega_{sHt}\hat{\hat{U}}_{sHt} - \iota_{Ht}\right) \\ \frac{1}{T}\sum_t \left(\sum_s \omega_{s1t}\left(X^{V\prime}_{s1t}\beta_V - \delta^V_{1t}\right) - \sum_s \omega_{s1t}\hat{\hat{V}}_{s1t} - \iota_{1t}\right) \\ \vdots \\ \frac{1}{T}\sum_t \left(\sum_s \omega_{sHt}\left(X^{V\prime}_{sHt}\beta_V - \delta^V_{Ht}\right) - \sum_s \omega_{sHt}\hat{\hat{V}}_{sHt} - \iota_{Ht}\right) \end{pmatrix}.$$

Our estimator is the minimum distance estimaor where the moment condition is specifed in (5.2.2). When we write the asymptotic variance of $\hat{\hat{U}}_{sht}$ and $\hat{\hat{V}}_{sht}$ by S^U_t and S^V_t, the optimally weighted minimum distance estimator is defined as follows:

$$\hat{\theta} \equiv \arg\min_{\theta}\ g'(\theta)S^{-1}g(\theta),$$

where

$$S = \begin{pmatrix} \frac{1}{T^2}\sum_t S^U_t & 0 \\ 0 & \frac{1}{T^2}\sum_t S^V_t \end{pmatrix}.$$

We can compute the asymptotic distribtuion of the estimator as follows and the standard error can be obtained directly[43]. As the Poisson regression in the first step has the explicit form of S^U_t and S^V_t, by inserting the estimated results, we directly compute

[43]We assume that the polynomial approximation regarding the systematic utility is correct. When there is a misspecification, we must treat the bias due to the misspecified model, whichi is beyond the scope of this study.

get the estimates of the standard errors for every parameters.

Theorem 2. *Under the regularity conditions, the asymptotic distribution of $\hat{\theta}$ is as follows:*

$$\sqrt{ST}\left(\hat{\theta}-\theta\right) \xrightarrow{d} N\left(0, \left(\Gamma' S^{-1}\Gamma\right)^{-1}\right),$$

where $\Gamma = \frac{\partial}{\partial\theta} g(\theta)$.

C Additional analysis

C.1 Test of implicit tax on urban areas

Under the excessive competition for the slots in urban counties, it is possible that the surplus generated by matches in urban counties has already been diminished due to some external forces: for example, as the number of slots decreases, it becomes more difficult for residency programs to secure funding. Given this consideration, the marginal effect estimates for locations in Tokyo or other urban areas may be underestimated. Here, we examine whether matches in urban counties are subject to an implicit tax under the current market outcome.

Empirical strategy We define an individual level transfer as in the main analysis. Fix any period t and an implicit taxation policy $w_t = (w_{zt})_z$. We define *individual-level transfer from hospital h to doctor i with a ratio α*, denoted by τ_{iht}, as follows:

$$\tau_{iht} \coloneqq u_{it} - \left(U^{\text{base}}_{sht} + \varepsilon_{iht} - \alpha w_{zt}\right).$$

Tax w_{zt} is levied on the matched pair of doctor i and hospital h. The doctor incurs fraction α of the tax; thus doctor's payoff *without transfer* were to be $U^{\text{base}}_{ijt} - \alpha w_{rt} = U^{\text{base}}_{sht} + \varepsilon_{iht} - \alpha w_{rt}$. In equilibrium, doctor i enjoys equilibrium payoff u_{it}, which could be different from U^{base}_{ijt}. We interpret the difference between equilibrium payoff and payoff without transfer as the individual-level transfer from the hospital side to the doctor side.

Now we define an *aggregate-level transfer* as the average of the individual-level transfer in a hospital h and denote it by ι_{ht}:

$$\iota_{ht} \coloneqq \frac{1}{|D(h)_t|} \sum_{i \in D(h)_t} \tau_{iht},$$

where $D(h)_t$ is the set of doctors matched with any slot of hospital h at time t. We have the same moment conditions for this case as in the main analysis.

As in the main analysis, we model the base utilities as a linear function of observable characteristics. In addition to them, we define $\delta^U_{ht} \equiv \alpha w_{r(h)t}$ and $\delta^V_{ht} \equiv (1-\alpha) w_{r(h)t}$ as the levied implicit tax on school side and hospital side in period t and treat them as parameters to be estimated. Our parameters of interest are the following: β_U, β_V, δ^U_{ht} for every pair of h and t, and δ^V_{ht} for every pair of z and t. We use θ to indicate the vector of these parameters: $\theta \coloneqq \left(\beta_U, (\delta^U_{ht})_{h,t}, \beta_V, (\delta^V_{ht})_{h,t}\right)$.

For the estimation of the aggregate-level utilities are same as our main analysis. For the second step, we construct the following moment conditions for θ:

$$\sum_s \omega_{sht} \left(X^{U,\text{base}\prime}_{sht} \beta_U - \delta^U_{ht} \right) = \sum_s \omega_{sht} \hat{U}_{sht} - \iota_{ht}, \ \forall \ h, t$$
$$\sum_s \omega_{sht} \left(X^{V,\text{base}\prime}_{sht} \beta_V - \delta^V_{ht} \right) = \sum_s \omega_{sht} \hat{V}_{sht} + \iota_{ht}, \ \forall \ h, t$$

By adopting the same measurement model, the estimating equations are as follows:

$$\sum_s \omega_{sht} \hat{U}_{sht} = \gamma_{0,U} + \gamma_{1,U} S_{ht} + \sum_s \omega_{sht} \left(X^{U\prime}_{sht} \beta_U - \delta^U_{ht} \right) + \psi^U_{ht}$$
$$\sum_s \omega_{sht} \hat{V}_{sht} = \gamma_{0,V} + \gamma_{1,V} S_{ht} + \sum_s \omega_{sht} \left(X^{V\prime}_{sht} \beta_V - \delta^V_{ht} \right) + \psi^V_{ht}.$$

We estimate these linear equations using BLP-type IVs.

Estimation results We take advantage of the fact that the regional constraints on urban areas are getting the more strict as time goes to clarify the existence of implicit tax. As we explain in Section 2, the government lowers the upper bounds on the number of matches in the urban areas by 5% every year. Hence, if the surplus in urban areas have been decreased due to the constraints, the estimated coefficients on dummy variables of urban or Tokyo will decrease over the years.

Table 11 shows the estimation results based on IV estimation: where we include all the

Table 11. Estimation Result: Tax Parameters
Degree of polynomials = 3

	(1) University	(2) University	(3) Hospital	(4) Hospital
Constant	-6.658***	-6.704***	1.875**	1.966**
	(0.314)	(0.327)	(0.848)	(0.874)
Salary (million Yen)	2.412***	2.519***	-1.579**	-1.810**
	(0.445)	(0.478)	(0.706)	(0.793)
Urban	0.0338	0.0429	0.114*	0.119*
	(0.0470)	(0.0496)	(0.0671)	(0.0702)
Urban × 2018	0.0639	-0.00972	0.0882	0.0805
	(0.0577)	(0.0644)	(0.0786)	(0.0878)
Urban × 2019	0.0752	0.0509	-0.0886	-0.0634
	(0.0575)	(0.0640)	(0.0821)	(0.0892)
Tokyo		-0.00704		-0.0740
		(0.0751)		(0.116)
Tokyo × 2018		0.265***		0.0282
		(0.102)		(0.144)
Tokyo × 2019		0.0903		-0.0940
		(0.102)		(0.155)
N	2627	2627	2627	2627
Other covariates	√	√	√	√
Tokyo × Year		√		√

Standard errors in parentheses
* p<0.1, ** p<0.05, *** p<0.01

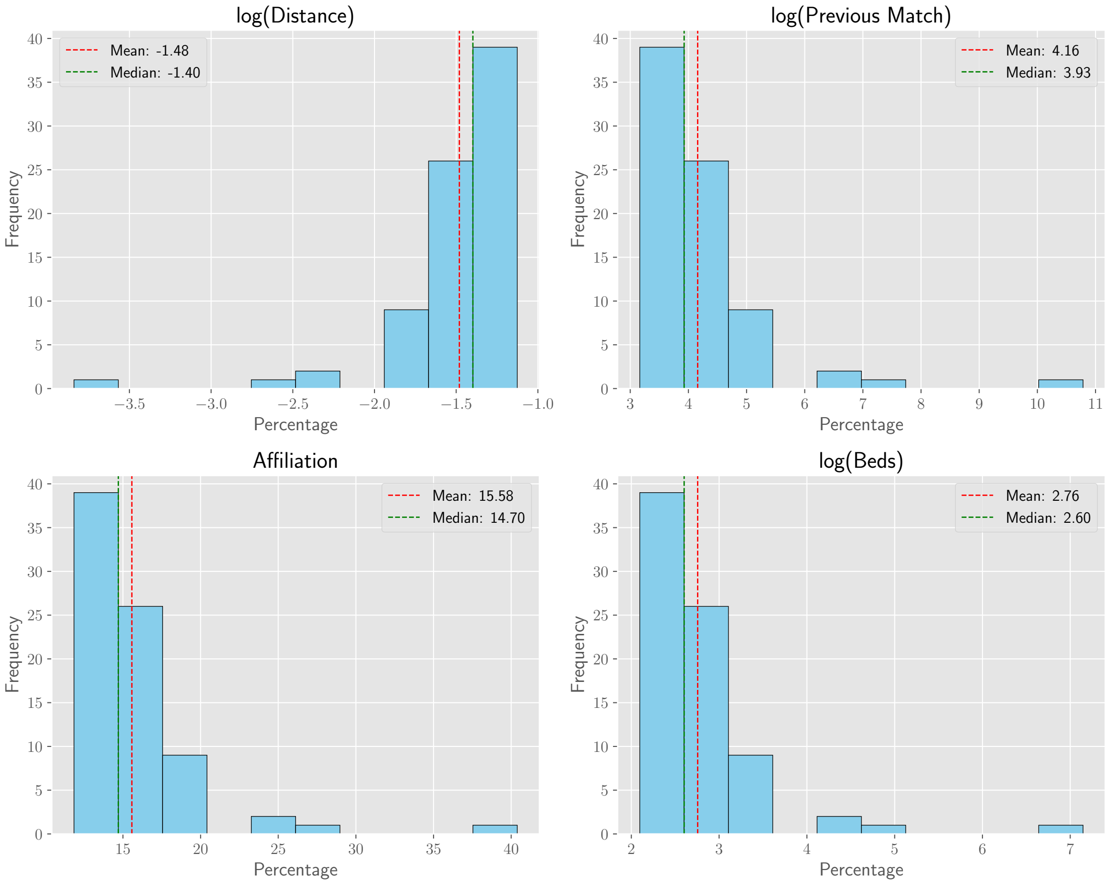


Figure 8. Relative Size of Coefficients in Doctor's Preference.

covariates in Table 5 and additioanly the interaction terms between dummy variables of urban and Tokyo and the dummy variables of each year. As found in every specifications for both sides, we do not find the decrease in the coefficients of dummy variables of urban areas. Furthermore, we do not find any positive impact of living in urban areas except for living in Tokyo in 2018. Based on these results, we conclude that the current market does not suffer from any implicit tax and the market outcome is the aggregate equilibrium under the reduced capacities.

C.2 Relative impacts

To grab the sizes of impacts, we compute the ratio of these coefficients to the aggregate-level utility. Specifically, we first transform the aggregate-level utilities into monetary unit based on the estimation results in the second stage estimation: for doctor side, we

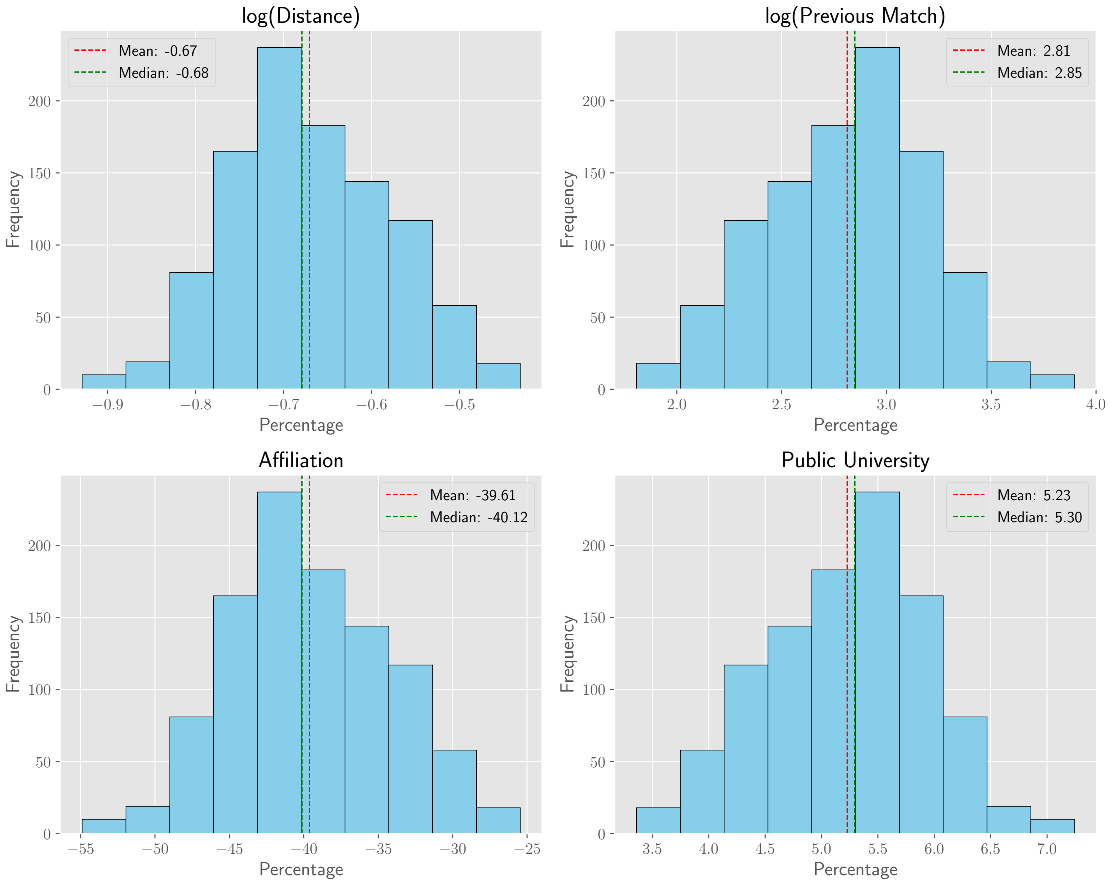


Figure 9. Relative Size of Coefficients in Hospital's Preference.

compute $U^{money}_{sht} \equiv \frac{\hat{\hat{U}}_{sht} - \hat{\gamma}_{0,U,t}}{\hat{\gamma}_{1,U}}$ and for hospital side, we compute $V^{money}_{sht} \equiv \frac{\hat{\hat{V}}_{sht} - \hat{\gamma}_{0,V,t}}{\hat{\gamma}_{1,V}}$.[44] Then, we take the average of these transformed aggregate-level utilities with respect to the periods and the institutions in the other side of the market: $\bar{U}^{money}_{s} \equiv \frac{1}{HT}\sum_{h,t} U^{money}_{sht}$ and $\bar{V}^{money}_{h} \equiv \frac{1}{ST}\sum_{s,t} V^{money}_{sht}$. $\bar{U}^{money}_{s}$ and $\bar{V}^{money}_{s}$ are measures of the expected utilities in the matching market computed for every universities and hospitals. Finally, we take the ratio of the estimated coefficients to these measures to grab the relative size of the coefficients. For the logarithm covariates, we compute the relative size of 10% changes of the covariates.

Figure 8 depicts the histograms of the relative size of the coefficients in doctor's preference for the four covariates which have statistically significant influence in Table 6: the logarithm of distance, the logarithm of the number of previous matches, the dummy

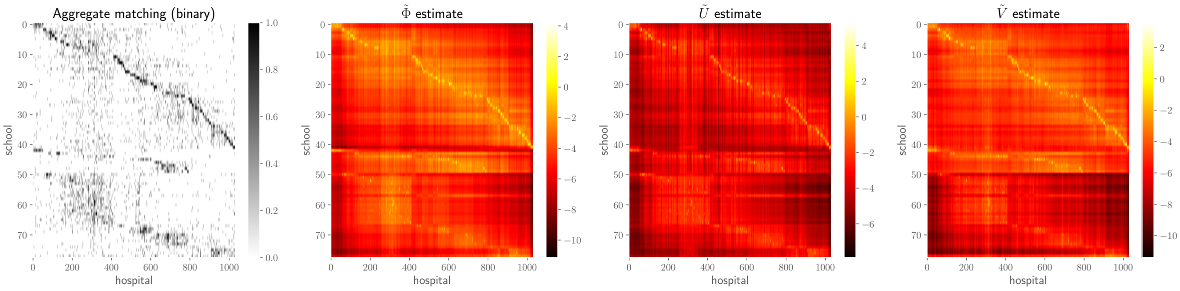
[44] Note that the constants depend on period t, i.e. $\hat{\gamma}_{0,U,t}$ and $\hat{\gamma}_{0,V,t}$, because we include the dummy variables of every years.

variable of affiliation, and the logarithm of the number of beds. In each panel, we show the mean and the median of the relative size of impacts. Although there is variation in the utility level among the universities, the distribution of the relative size of impacts has single peak and their means and the medians are not so different. The average of the relative size of impact of 10% change in distance amounts to 1.4% of the doctor's utility, the same one of the number of previous match amounts to 4.16%, and the same one of the number of beds amounts to 2.76%. On average, affiliation relationship amounts to about 15.58% of the doctors' average utillities.

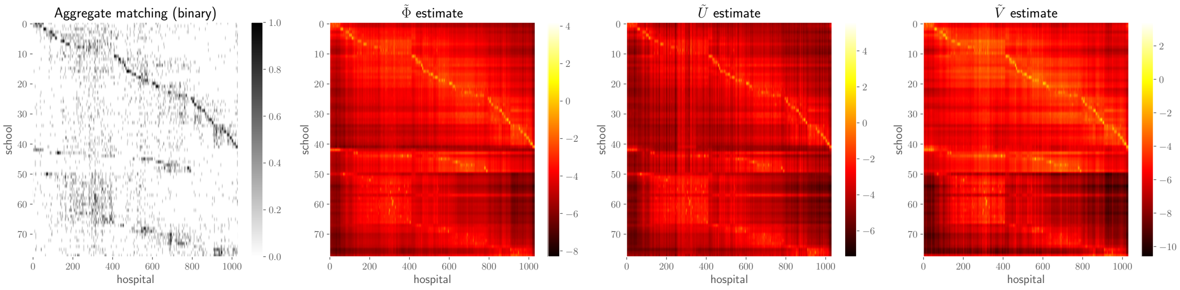
Figure 9 shows the same histograms for the hospital side preference. We plot the historgrams of the four covariates which shows the statistically significance in Table 7: the logarithm of distance, the logarithm of the number of previous matches, affiliation relationship, and the indicator of the public university. As the average utilities of hospitals are larger than the ones of doctors in the monetary unit sense, the computed relative size of impacts are likely smaller than the values obtained in the case of doctors. The average of the relative size of impact of 10% change in distance amounts to 0.67% of the doctor's utility and the same one of the number of previous match amounts to 2.81%. On average, graduates from public university, which is usually an elite school, gives 5.23% increase in the utility of hospitals. Although the affiliation relationship gives the largest negative impact on the utility of hospitals, this estimate is not stable for the choice of the degree of polynomials as shown in Appendix C.3.

C.3 Results when the degree of polynomial is set to 2

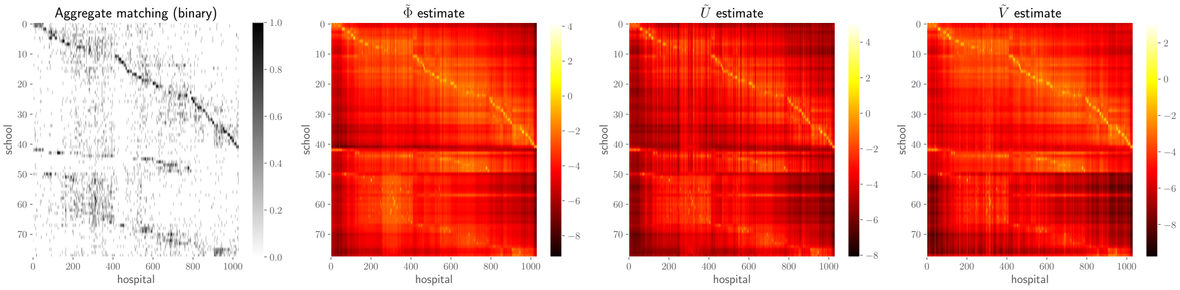
Here we show the empirical results obtained when we set the degree of polynomials in the first step to two. All the tables and figures listed here corresponds to the tables and figures shown in Section 6 and Appendix C.1. We do not find any qualittative difference in the main findings from the case where we set the degree of polynomials to three.



(a) 2017



(b) 2018



(c) 2019

Figure 10. Aggregate matchings, estimated systematic utilities and estimated social surpluses.

Table 12. Estimation Result: Preference Parameters
Degree of polynomials = 2

	(1) University	(2) University (IV)	(3) Hospital	(4) Hospital (IV)
Constant	-5.917***	-7.988***	1.485**	1.703**
	(0.225)	(0.429)	(0.705)	(0.787)
Salary (million Yen)	0.564***	4.129***	0.704***	-1.231*
	(0.143)	(0.627)	(0.142)	(0.718)
Tokyo	-0.129***	0.0102	0.100*	-0.00294
	(0.0492)	(0.0604)	(0.0524)	(0.0621)
urban	-0.102***	0.0524	0.252***	0.191***
	(0.0339)	(0.0436)	(0.0317)	(0.0418)
log(Distance)	-0.380***	-0.400***	-0.331***	-0.304***
	(0.0157)	(0.0158)	(0.0150)	(0.0172)
log(Previous Match)	1.583***	1.563***	1.663***	1.667***
	(0.0398)	(0.0302)	(0.0398)	(0.0449)
Affiliation	-0.488**	-0.431***	-2.676***	-2.827***
	(0.199)	(0.146)	(0.173)	(0.194)
University hospital	-0.199**	0.0127		
	(0.0800)	(0.103)		
Govermental hospital	0.0319	-0.0589		
	(0.0341)	(0.0433)		
log(Beds)	0.511***	0.628***		
	(0.0359)	(0.0462)		
Public university			0.182***	0.176***
			(0.0531)	(0.0573)
Prestige			-1.905***	-3.125***
			(0.668)	(0.727)
N	2847	2627	2847	2627

Standard errors in parentheses
* p<0.1, ** p<0.05, *** p<0.01

Table 13. Estimation Result: Tax Parameters
Degree of polynomials = 2

	(1) University	(2) University	(3) Hospital	(4) Hospital
Constant	-8.028***	-7.928***	1.755**	1.725**
	(0.415)	(0.427)	(0.763)	(0.784)
Salary (million Yen)	4.298***	4.121***	-1.276**	-1.234*
	(0.589)	(0.625)	(0.643)	(0.719)
Urban	-0.100	-0.116*	0.106*	0.0755
	(0.0621)	(0.0649)	(0.0623)	(0.0646)
Urban × 2018	0.275***	0.247***	0.179**	0.239***
	(0.0763)	(0.0842)	(0.0720)	(0.0811)
Urban × 2019	0.217***	0.256***	0.0655	0.107
	(0.0761)	(0.0838)	(0.0749)	(0.0805)
Tokyo		0.0257		0.119
		(0.0982)		(0.101)
Tokyo × 2018		0.0976		-0.216*
		(0.133)		(0.123)
Tokyo × 2019		-0.145		-0.151
		(0.133)		(0.132)
N	2627	2627	2627	2627
Other covariates	√	√	√	√
Tokyo × Year		√		√

Standard errors in parentheses
* p<0.1, ** p<0.05, *** p<0.01

Table 14. University preference parameters (unit: million Yen)
Degree of polynomials = 2

	(1)	(2)	(3)
Coefficient of Salary =	4.129	4.298	4.121
log(Distance)	-0.097***	-0.094***	-0.097***
	(0.01)	(0.01)	(0.01)
log(Previous Match)	0.378***	0.363***	0.379***
	(0.06)	(0.05)	(0.06)
Affiliation	-0.104**	-0.102**	-0.106**
	(0.04)	(0.04)	(0.04)
University Hospital	0.003	0.006	0.003
	(0.02)	(0.02)	(0.02)
Governmental Hospital	-0.014	-0.015	-0.014
	(0.01)	(0.01)	(0.01)
log(Beds)	0.152***	0.148***	0.153***
	(0.02)	(0.02)	(0.02)
N	2627	2627	2627
Urban $\times$ Year		$\surd$	$\surd$
Tokyo $\times$ Year			$\surd$

Table 15. Hospital preference parameters (unit: million Yen)
Degree of polynomials = 2

	(1)	(2)	(3)
Coefficient of Salary =	1.231	1.276	1.234
log(Distance)	-0.247	-0.238	-0.247
	(0.15)	(0.13)	(0.15)
log(Previous Match)	1.354	1.307*	1.352
	(0.79)	(0.66)	(0.79)
Affiliation	-2.296	-2.222*	-2.294
	(1.29)	(1.07)	(1.29)
Public University	0.143	0.136	0.141
	(0.09)	(0.07)	(0.09)
Prestige	-2.537	-2.451*	-2.522
	(1.37)	(1.15)	(1.36)
N	2627	2627	2627
Urban × Year		√	√
Tokyo × Year			√

C.4 Counterfactual simulations for the other years

Table 16. Comparison between Aggregate-level Equilibria

Equilibrium	**AE**	**AE**	**EAE**
Capacity	Reduced	True	True
Floor	No	No	Yes
2017			
Match rate	0.868	0.912	0.912
Doctors' welfare	82874.9	84507.3	84514.3
Hospitals' welfare	51788.4	56209.0	56211.2
Government's revenue	0.0	0.0	$[-10.5, -7.4]$
Total welfare	134663.3	140716.3	$[140715.0, 140718.1]$
#(subsidized regions)	0	0	3
Average subsidy	0.000	0.000	−0.040
#(constraint violations)	0	3	0
2018			
Match rate	0.844	0.895	0.896
Doctors' welfare	85736.8	86606.8	86618.1
Hospitals' welfare	53076.2	59986.4	59992.0
Government's revenue	0.0	0.0	$[-18.9, -13.3]$
Total welfare	138812.9	146593.2	$[146591.3, 146596.8]$
#(subsidized regions)	0	0	5
Average subsidy	0.000	0.000	−0.038
#(constraint violations)	0	5	0
2019			
Match rate	0.869	0.912	0.912
Doctors' welfare	84009.3	85291.8	85300.2
Hospitals' welfare	53255.4	58114.9	58115.4
Government's revenue	0.0	0.0	$[-9.4, -6.6]$
Total welfare	137264.7	143406.6	$[143406.2, 143409.0]$
#(subsidized regions)	0	0	2
Average subsidy	0.000	0.000	−0.042
#(constraint violations)	0	2	0

* All values except match rates, #(subsidized regions), and #(constraint violations) are expressed in units of 1 million JPY per month. The government's revenue is positive when taxes are imposed on doctors and hospitals and negative when subsidies are provided to them. The welfare of doctors and hospitals is scaled according to specification (1) in Table 6 and Table 7. We present the bounds of the government's net revenue, scaled by the coefficients on the doctor side and the hospital side, respectively. The total welfare is the sum of doctors' welfare, hospitals' welfare, and the government's revenue. #(constraint violations) counts the number of prefectures violating the lower bounds (among the 15 rural regions).